

Introductory Workshop on Time Series Analysis

Sara McLaughlin Mitchell
Department of Political Science
University of Iowa

Overview

- Properties of time series data
- Approaches to time series analysis
- Stationarity and unit roots
- Advanced topics
 - Cointegration
 - Error Correction Models (ECM)

What is time series data?

- A time series is a collection of data y_t ($t=1,2,\dots,T$), with the interval between y_t and y_{t+1} being fixed and constant.
- We can think of time series as being generated by a stochastic process, or the data generating process (DGP).
- A time series (sample) is a particular realization of the DGP (population).
- Time series analysis is the estimation of difference equations containing stochastic (error) terms (Enders 2010).

Types of Time Series Data

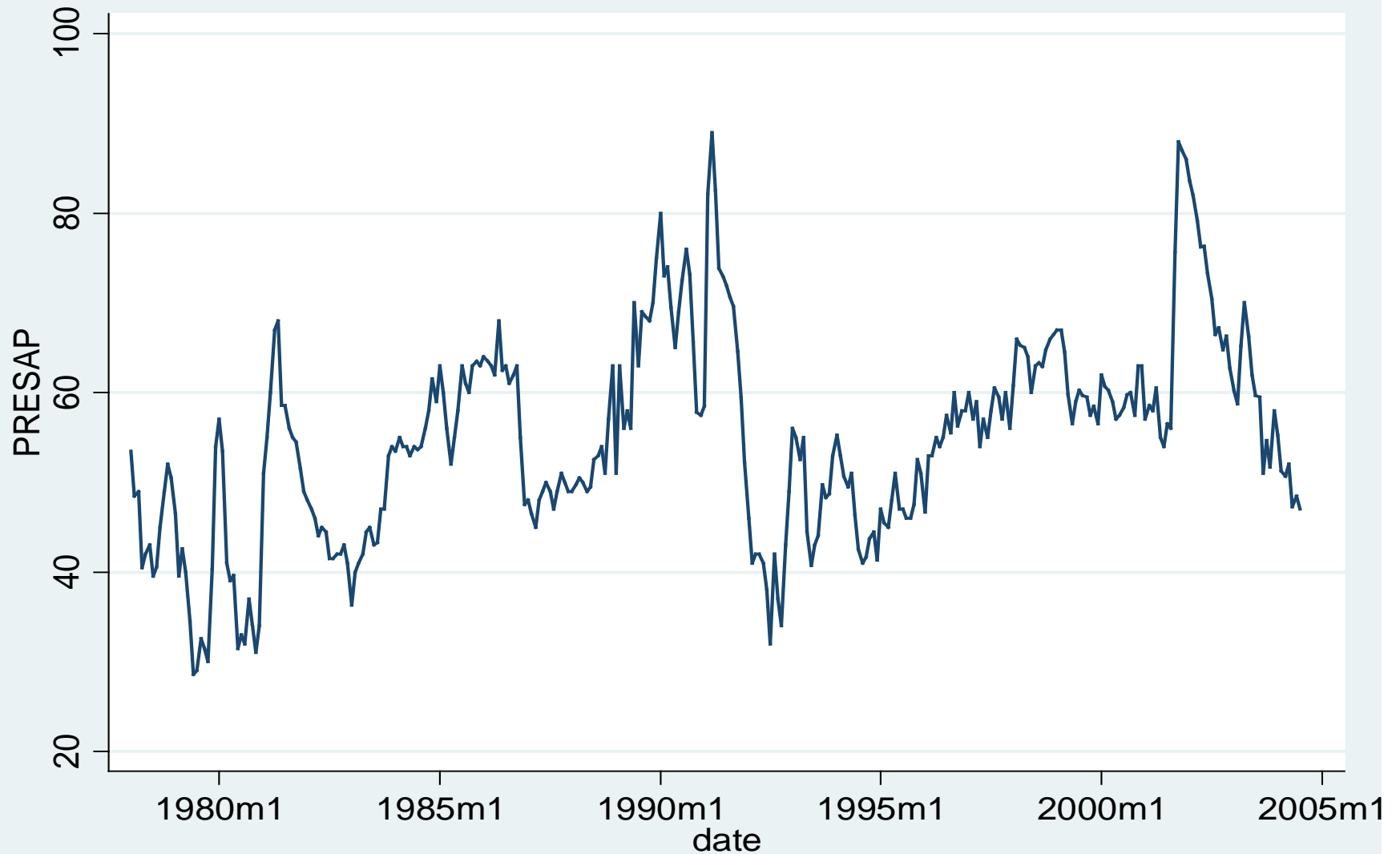
- Single time series
 - U.S. presidential approval, monthly (1978:1-2004:7)
 - Number of militarized disputes in the world annually (1816-2001)
 - Changes in the monthly Dow Jones stock market value (1978:1-2001:1)
 - Number of homicides per month in the U.S.
- Pooled time series
 - Dyad-year analyses of interstate conflict
 - State-year analyses of welfare policies
 - Country-year analyses of economic growth

Properties of Time Series Data

- Property #1: Time series data have autoregressive (AR), moving average (MA), and seasonal dynamic processes.
- Because time series data are ordered in time, past values influence future values.
- This often results in a violation of the assumption of no serial correlation in the residuals of a standard OLS model.

$$\text{Cov}[\varepsilon_i, \varepsilon_j] = 0 \text{ if } i \neq j$$

U.S. Monthly Presidential Approval Data, 1978:1-2004:7



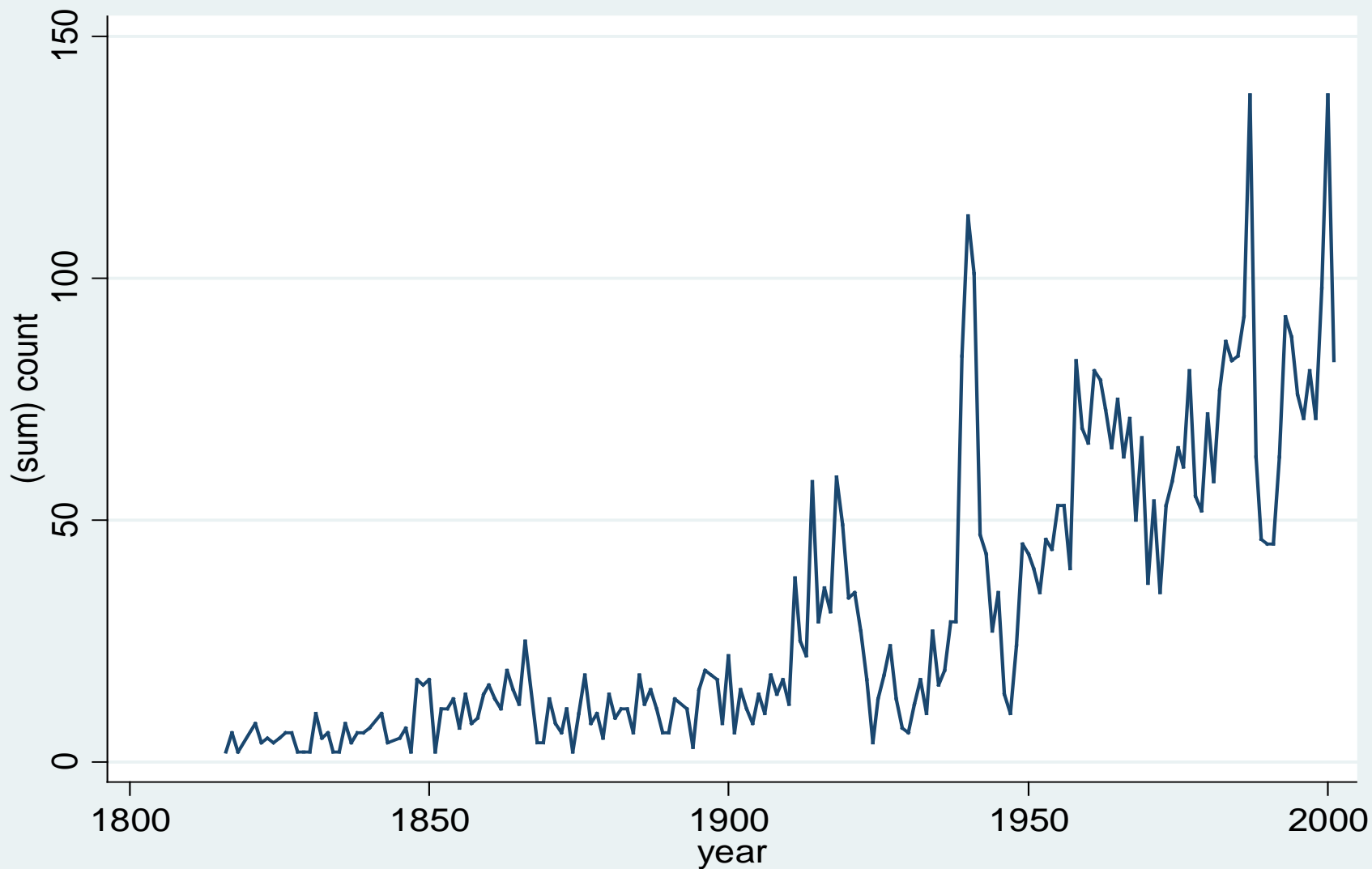
OLS Strategies

- When you first learned about serial correlation when taking an OLS class, you probably learned about techniques like generalized least squares (GLS) to correct the problem.
- This is not ideal because we can improve our explanatory and forecasting abilities by modeling the dynamics in Y_t , X_t , and ε_t .
- The naïve OLS approach can also produce spurious results when we do not account for temporal dynamics.

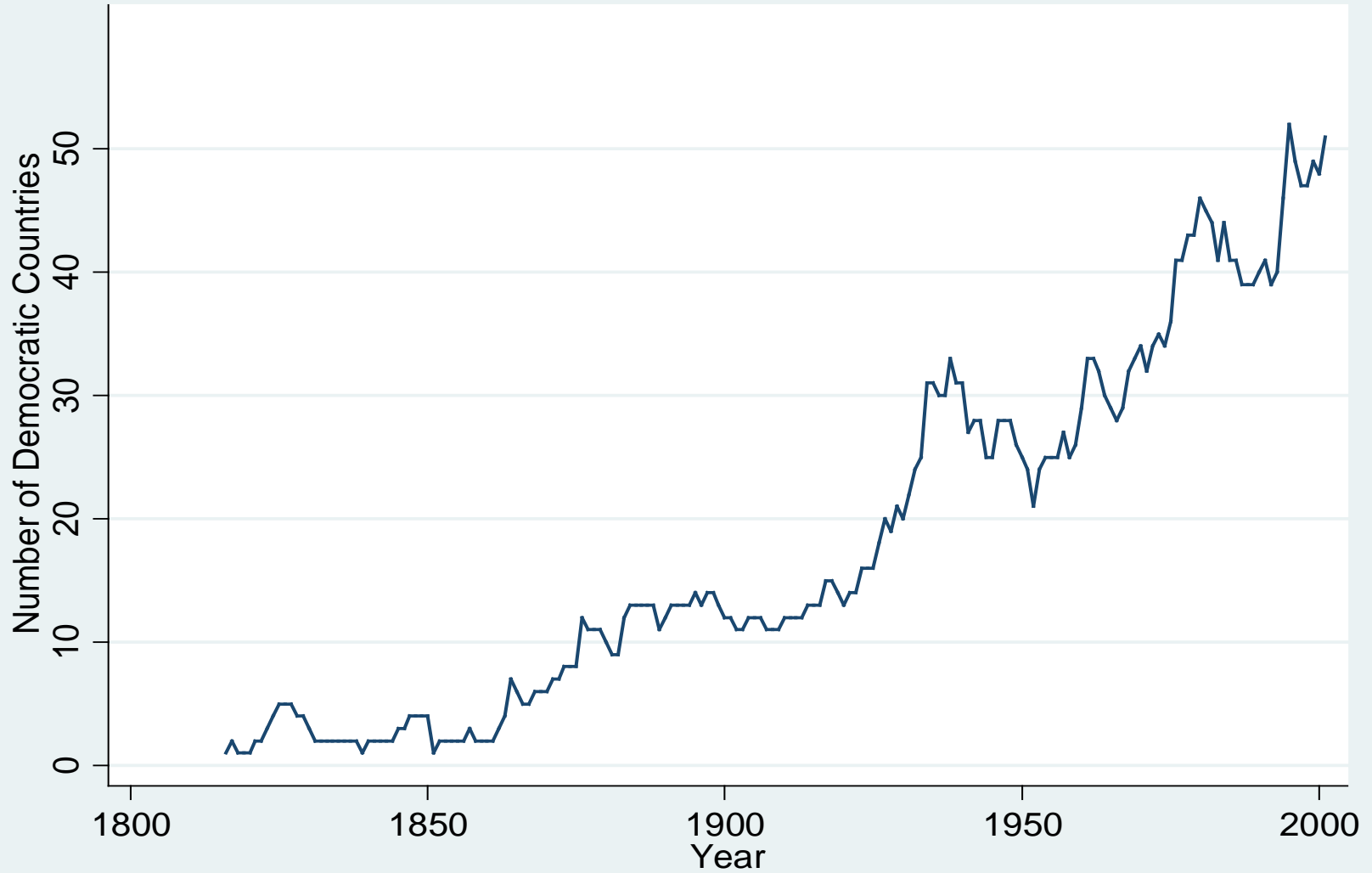
Properties of Time Series Data

- Property #2: Time series data often have time-dependent moments (e.g. mean, variance, skewness, kurtosis).
- The mean or variance of many time series increases over time.
- This is a property of time series data called nonstationarity.
- As Granger & Newbold (1974) demonstrated, if two independent, nonstationary series are regressed on each other, the chances for finding a spurious relationship are very high.

Number of Militarized Interstate Disputes (MIDs), 1816-2001



Number of Democracies, 1816-2001

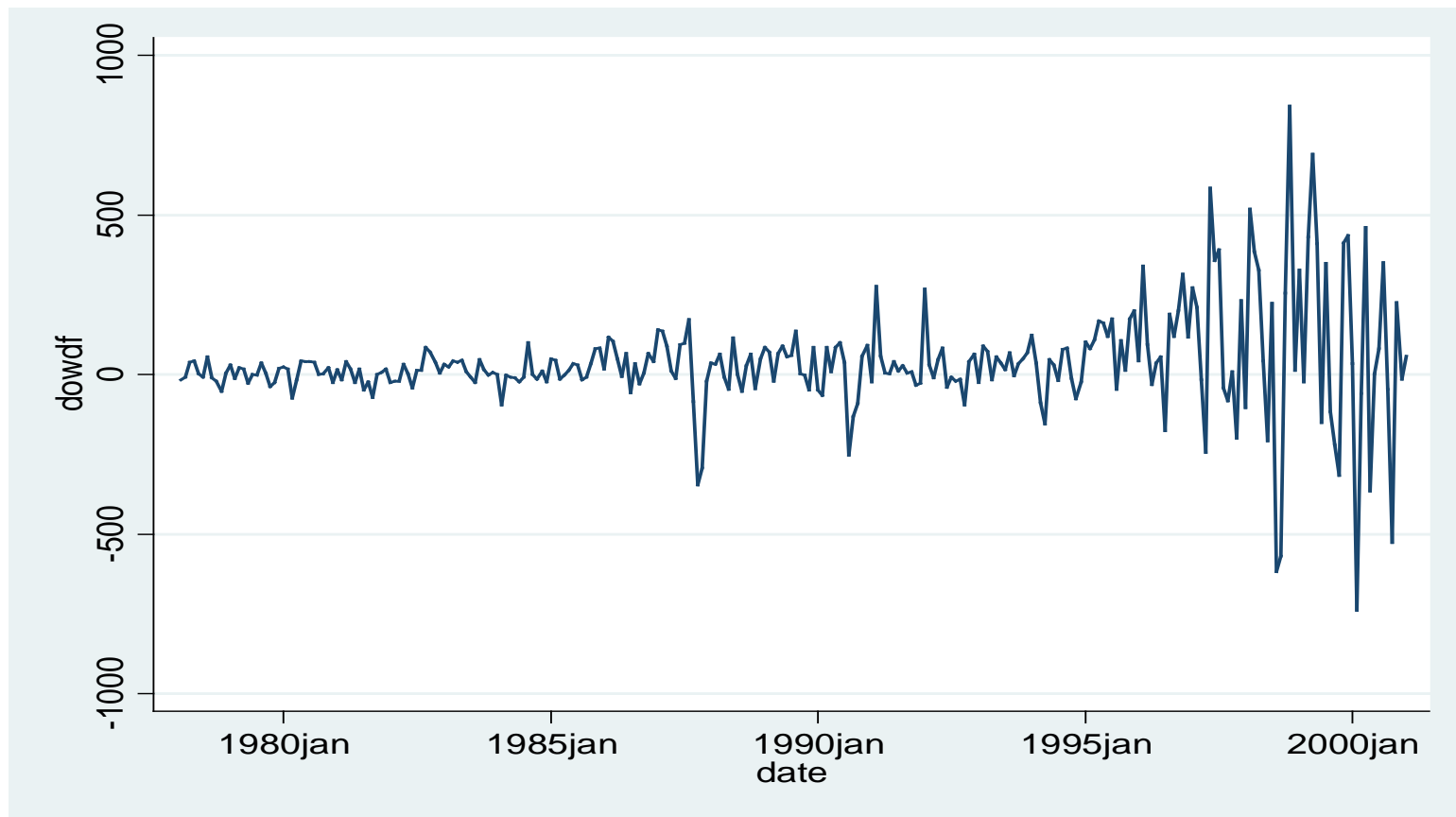


Democracy-Conflict Example

- We can see that the number of militarized disputes and the number of democracies is increasing over time.
- If we do not account for the nonstationarity of each time series, we could erroneously conclude that more democracy causes more conflict.
- These series also have significant changes over time (WWII, end of Cold War), which could alter the observed X-Y relationship.

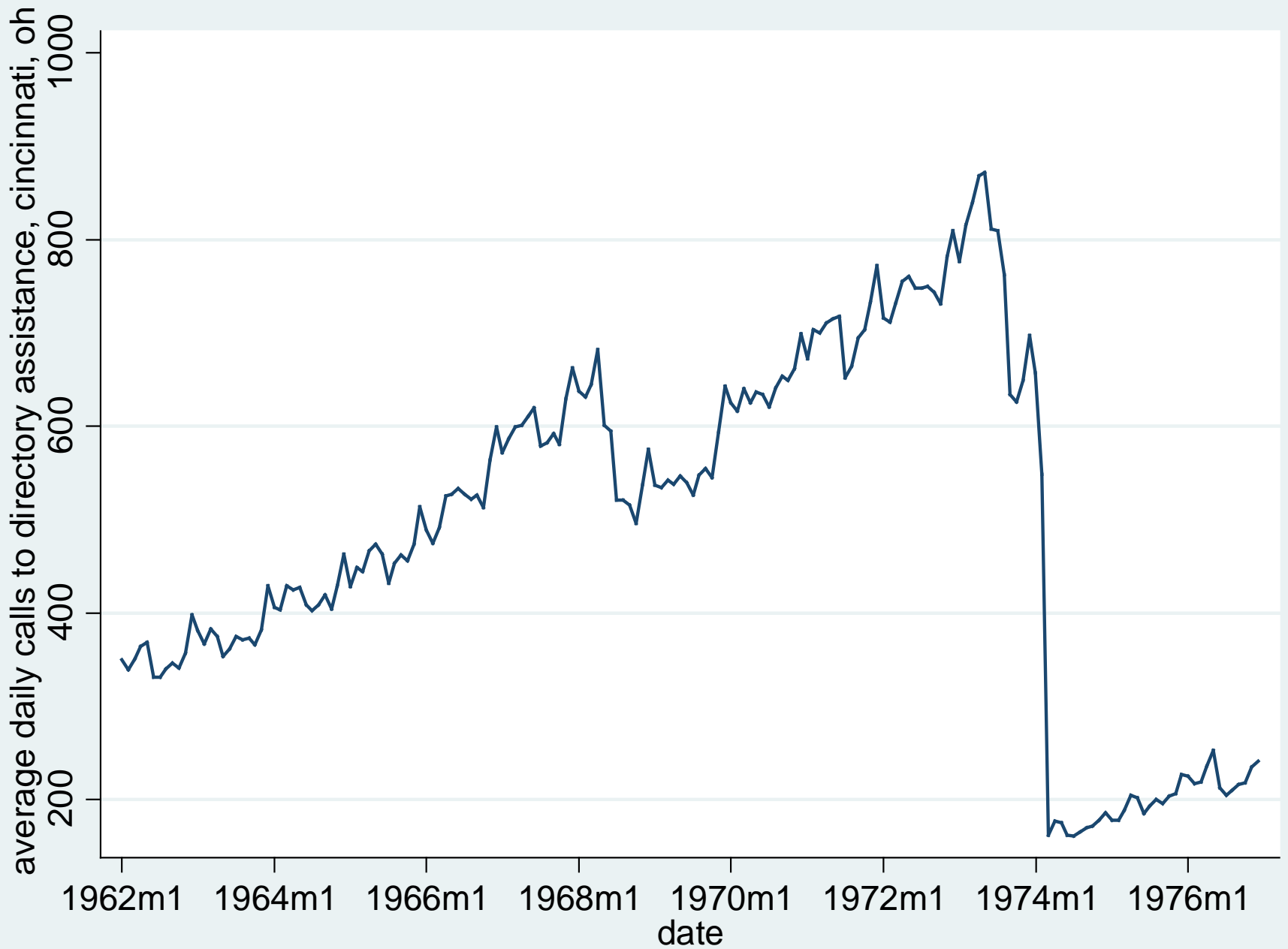
Nonstationarity in the Variance of a Series

- If the variance of a series is not constant over time, we can model this heteroskedasticity using models like ARCH and GARCH.
- Example: Changes in the monthly DOW Jones value.



Properties of Time Series Data

- Property #3: The sequential nature of time series data allows for forecasting of future events.
- Property #4: Events in a time series can cause structural breaks in the data series. We can estimate these changes with intervention analysis, transfer function models, regime switching/Markov models, etc.



Properties of Time Series Data

- Property #5: Many time series are in an equilibrium relationship over time, what we call cointegration. We can model this relationship with error correction models (ECM).
- Property #6: Many time series data are endogenously related, which we can model with multi-equation time series approaches, such as vector autoregression (VAR) and vector error correction (VECM).
- Property #7: The effect of independent variables on a dependent variable can vary over time; we can estimate these dynamic effects with time varying parameter models.

Issues with OLS Time Series Models

- Spurious regression is very likely if two or more nonstationary series are regressed on each other.
- OLS estimates are sensitive to outliers.
- OLS attempts to minimize the sum of squares for errors; time series with a trend will result in OLS placing greater weight on the first and last observations.
- OLS treats the regression relationship as deterministic, whereas time series have many stochastic trends.
- We can do better modeling dynamics than treating them as a nuisance.

Regression Example, Approval

Regression Model: Dependent Variable is monthly US presidential approval, Independent Variables include unemployment (unempn), inflation (cpi), and the index of consumer sentiment (ics) from 1978:1 to 2004:7.

```
regress presap unempn cpi ics
```

Source	SS	df	MS	Number of obs = 319		
Model	9712.96713	3	3237.65571	F(3, 315)	=	33.69
Residual	30273.9534	315	96.1077885	Prob > F	=	0.0000
Total	39986.9205	318	125.745033	R-squared	=	0.2429
				Adj R-squared	=	0.2357
				Root MSE	=	9.8035

presap	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
unempn	-.9439459	.496859	-1.90	0.058	-1.921528	.0336359
cpi	.0895431	.0206835	4.33	0.000	.0488478	.1302384
ics	.161511	.0559692	2.89	0.004	.0513902	.2716318
_cons	34.71386	6.943318	5.00	0.000	21.05272	48.37501

Regression Example, Approval

Durbin's alternative test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	1378.554	1	0.0000

H0: no serial correlation

The null hypothesis of no serial correlation is clearly violated. What if we included lagged approval to deal with serial correlation?

```
. regress presap lagpresap unempn cpi ics
```

Source	SS	df	MS	Number of obs =	318
Model	34339.005	4	8584.75125	F(4, 313) =	475.91
Residual	5646.11603	313	18.0387094	Prob > F =	0.0000
				R-squared =	0.8588
				Adj R-squared =	0.8570
Total	39985.121	317	126.136028	Root MSE =	4.2472

presap	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lagpresap	.8989938	.0243466	36.92	0.000	.8510901	.9468975
unempn	-.1577925	.2165935	-0.73	0.467	-.5839557	.2683708
cpi	.0026539	.0093552	0.28	0.777	-.0157531	.0210609
ics	.0361959	.0244928	1.48	0.140	-.0119955	.0843872
_cons	2.970613	3.13184	0.95	0.344	-3.191507	9.132732

Durbin's alternative test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	4.977	1	0.0257

H0: no serial correlation

- We still have a problem with serial correlation and none of our independent variables has any effect on approval!
- Interpretation of the effect of the X's on approval must now take into account the lagged DV (the total multiplier).

Approaches to Time Series Analysis

- ARIMA/Box-Jenkins
 - Focused on single series estimation
 - Model a series' own dynamic properties
 - Can include intervention variables
 - Generalized version: transfer function models
 - Allows for estimation of the relationship between interval level X_t and Y_t time series
- Regression Analysis
 - Adapts OLS approach to take into account properties of time series
 - Estimate relationship between variables pre-whitened in the ARIMA process (e.g. Box-Steffensmeier, DeBoef, and Lin 2004)
 - Estimate general dynamic models (ADL, ECM) and test restrictions (DeBoef and Keele 2008)

Univariate Time Series Modeling Process

ARIMA (Autoregressive Integrated Moving Average)

$Y_t \rightarrow$ AR filter \rightarrow Integration filter \rightarrow MA filter

(long term) (stochastic trend) (short term)

$\rightarrow \varepsilon_t$
(white noise error)

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t + b_1 \varepsilon_{t-1}$$

ARIMA (2,0,1)

$$\Delta y_t = a_1 \Delta y_{t-1} + \varepsilon_t$$

ARIMA (1,1,0)

where $\Delta y_t = y_t - y_{t-1}$

General ADL Specification

- We can start with the Autoregressive Distributed Lag (ADL) model and test restrictions on it.

$$Y_t = \alpha_0 + \sum_{i=1}^p \alpha_i Y_{t-i} + \sum_{j=1}^n \sum_{i=0}^q \beta_{jp} X_{jt-i} + \varepsilon_t$$

- DeBoef & Keele use a simpler version:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \varepsilon_t$$

TABLE 1 Restrictions of the ADL General Dynamic Model

Type	ADL Model	Restriction
General	$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \varepsilon_t$	None
Partial Adjustment*	$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_0 X_t + \varepsilon_t$	$\beta_1 = 0$
Static ^a	$Y_t = \alpha_0 + \beta_0 X_t + \varepsilon_t$	$\alpha_1 = \beta_1 = 0$
Finite Distributed Lag ^b	$Y_t = \alpha_0 + \beta_0 X_t + \beta_1 X_{t-1} + \varepsilon_t$	$\alpha_1 = 0$
Differences ^c	$\Delta Y_t = \alpha_0 + \beta_0 \Delta X_t + \varepsilon_t$	$\alpha_1 = 1, \beta_0 = -\beta_1$
Dead Start	$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_1 X_{t-1} + \varepsilon_t$	$\beta_0 = 0$
Common Factor ^d	$Y_t = \beta_0 X_t + \varepsilon_t, \varepsilon_t = \beta_1 \varepsilon_{t-1} + u_t$	$\beta_1 = -\beta_0 \alpha_1$

* Also known as the Koyck model.

^a $k_1 = \beta_0$; Dynamic effects at lags beyond zero constrained to be zero.

^b $k_1 = \sum_{j=1}^n \sum_{i=0}^{q-1} \beta_{ji}$.

^c Infinite mean lag length.

^d $k_1 = \beta_0, \mu = 0$, EC rate 100%.

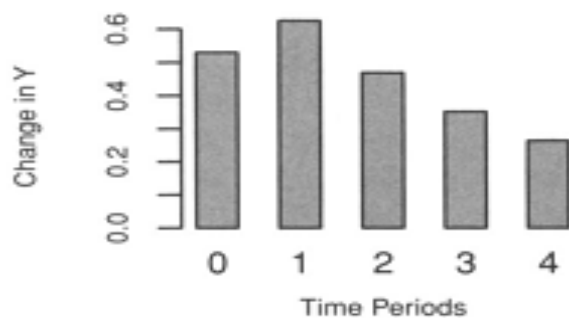
Multipliers

- For the basic ADL model in Table 1, the long run or total multiplier is given by:

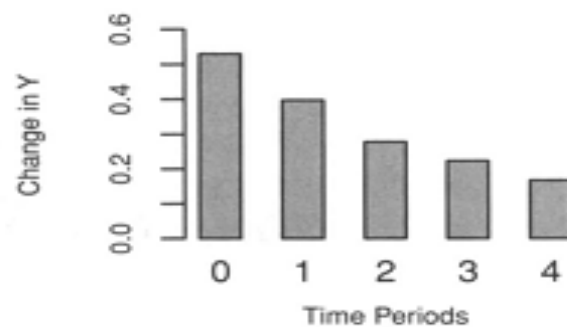
$$k_1 = \frac{(\beta_0 + \beta_1)}{(1 - \alpha_1)}$$

- Figure 1 shows different dynamic patterns.
- Costs of invalid restrictions
 - Can experience bias in coefficients and standard errors.

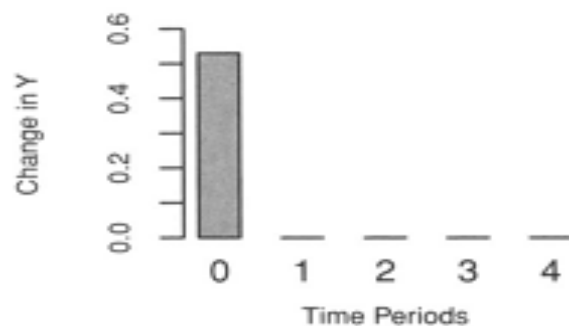
FIGURE 1 Simulated Lag Distributions: The Effect of Restrictions



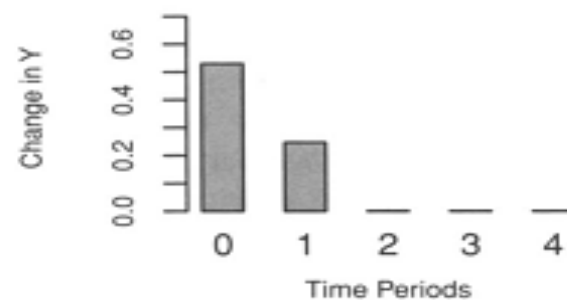
(a) Auto Distributed Lag Model



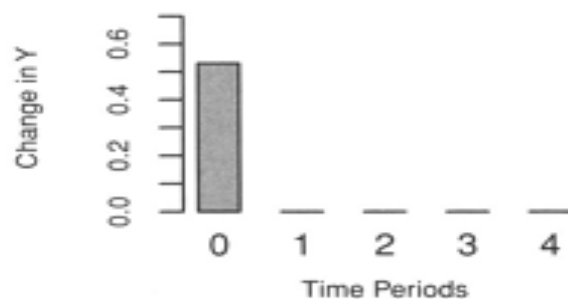
(b) Partial Adjustment Model, $\beta_1 = 0$



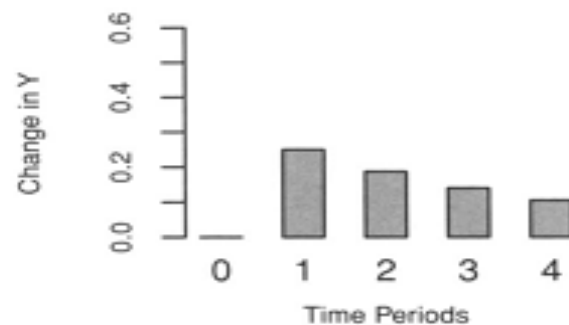
(c) Static Model, $\alpha_1 = \beta_1 = 0$



(d) Finite Distributed Lag, $\alpha = 0$



(e) Differences, $\alpha_1 = 1, \beta_0 = -\beta_1$



(f) Dead Start Model, $\beta_0 = 0$

Approaches to Time Series Analysis

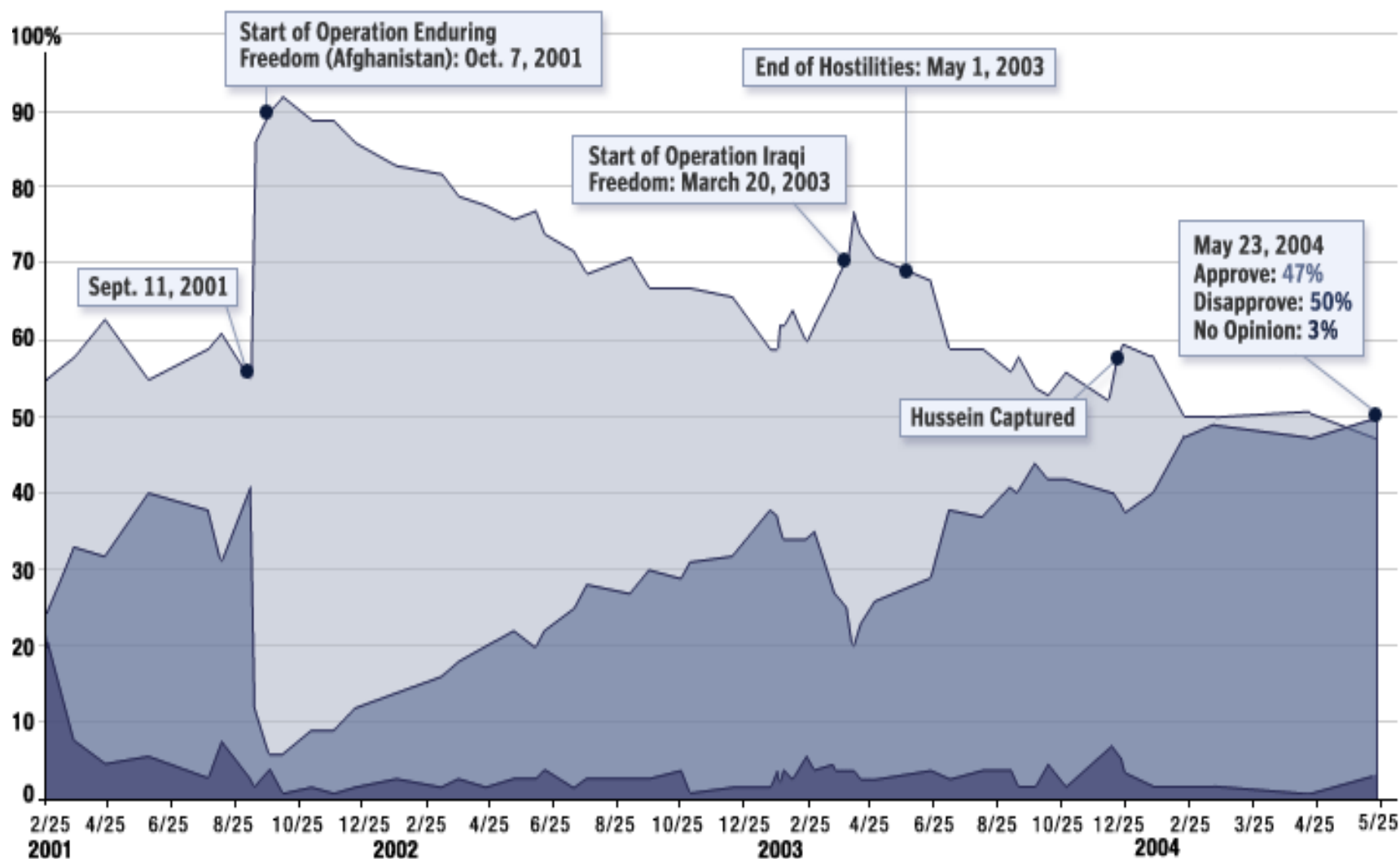
- London School of Economics (Granger, Hendry, Richard, Engle, etc.)
 - Combination of theory & empirics
 - Use theoretical models as guidelines for which variables to include in general specification
 - Move from general to specific models via testing
 - Error Correction Models (ECM)
- Minnesota (Sims)
 - Treats all variables as endogenous
 - Vector Autoregression (VAR)
 - Bayesian approach (BVAR); see also Leamer (EBA)

Stationarity: Shocks

- A *shock* is an event which takes place at a particular point in a time series.
 - Examples: effect of 9/2011 terrorist attack on Bush's approval; start of a new war; leader being overthrown, Watergate scandal
- Some shocks are modeled as interventions, while others are simply assumed to be exogenous to the model (e.g. captured in the error term).
- Shocks can alter:
 - The intercept
 - The slope
 - A model's dynamics (e.g. ARIMA process)
- We also introduce shocks in the residuals of a model to see how dynamics change (e.g. VAR).

Bush Approval Ratings

APPROVE DISAPPROVE NO OPINION



Source: A Washington Post/ABC News poll conducted by telephone May 20-23, 2004, among a national sample of 1,005 randomly selected adults. The margin of sampling error for overall results is plus or minus three percentage points.

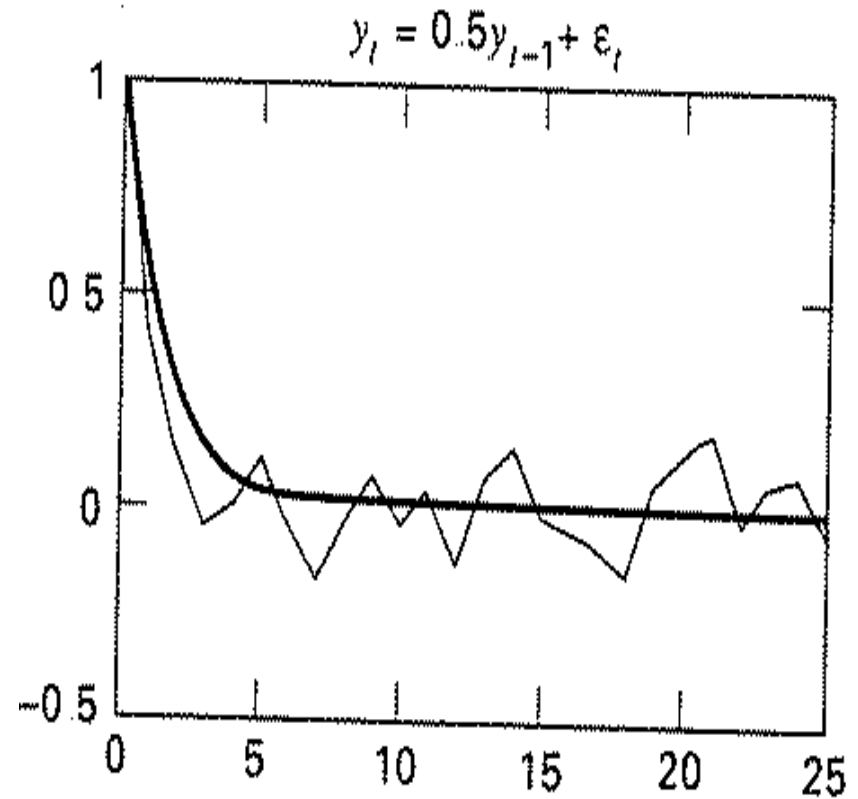
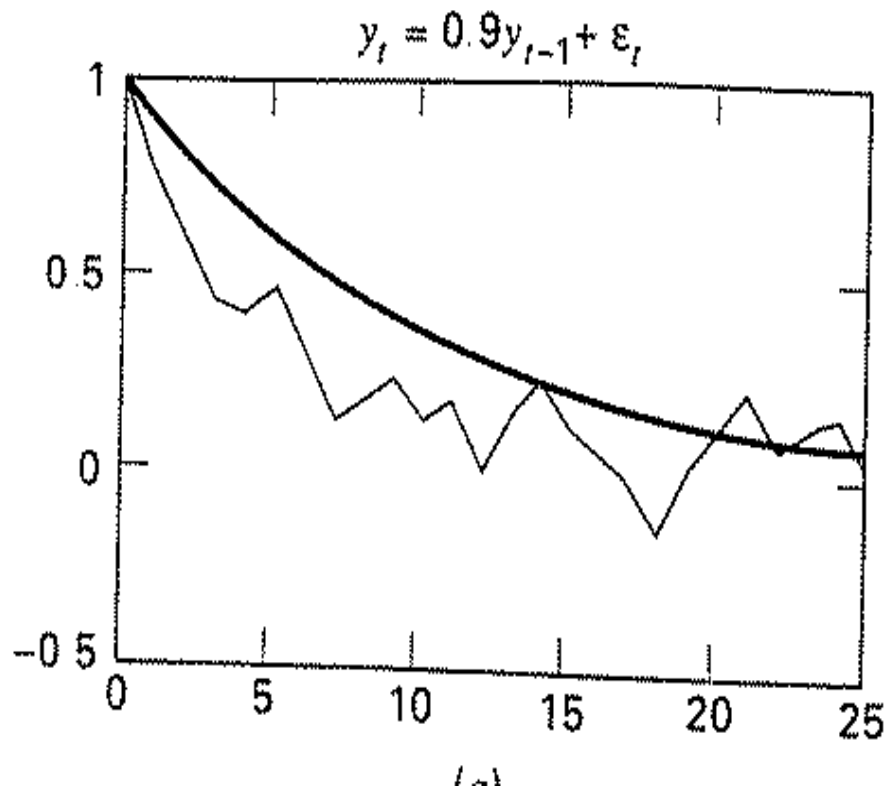
NOTE: Percentages might not add up to 100% due to rounding errors.

Stationary vs. Nonstationary Series

- Shocks to a stationary series are temporary; the series reverts to its long run mean.
- Shocks to a nonstationary series result in permanent moves away from the long run mean of the series.
- Stationary series have a finite variance that is time invariant; for nonstationary series, $\sigma^2 \rightarrow \infty$ as $t \rightarrow \infty$.
 - Problematic for statistical hypothesis testing

Convergence (mean = 0)

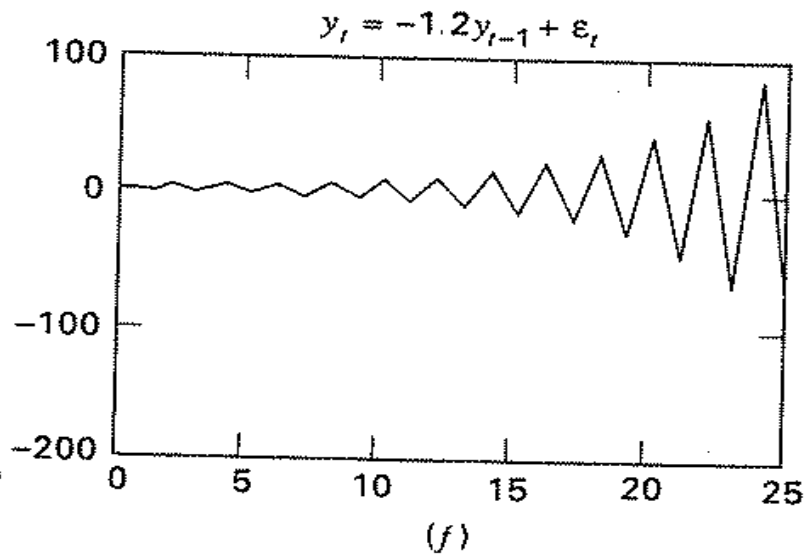
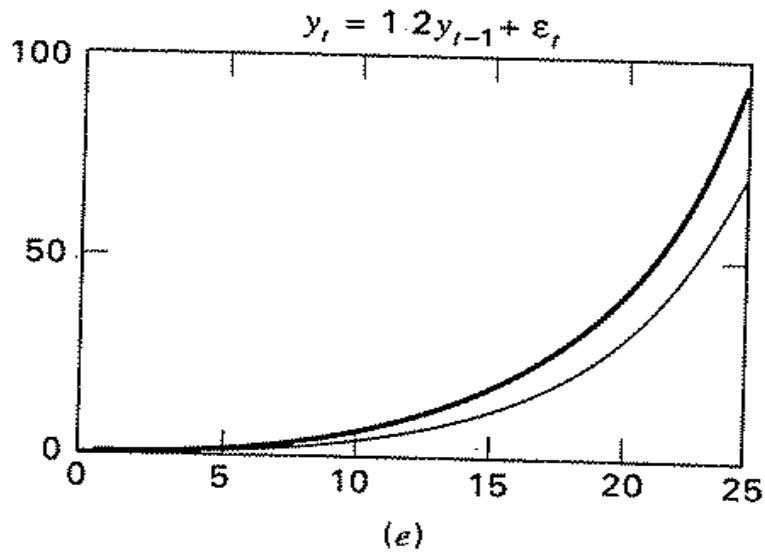
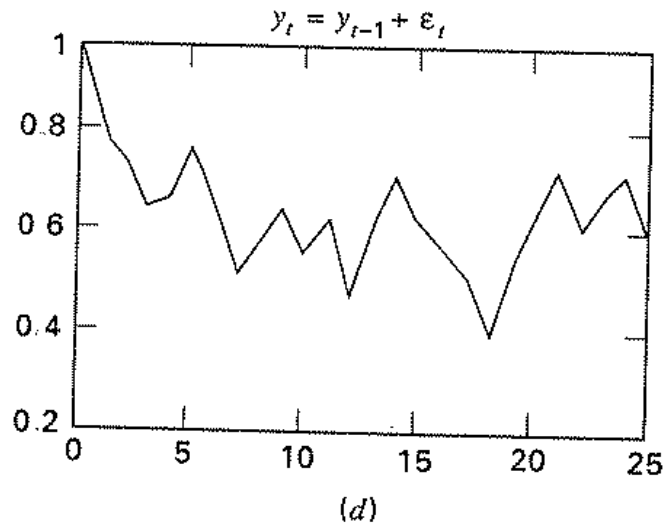
Figure 1.2 Convergent and nonconvergent s



(b)

Non-convergence (mean = 0)

Random walk



Unit Roots

- Consider an AR(1) model:

$$y_t = a_1 y_{t-1} + \varepsilon_t \quad (\text{eq. 1})$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

- Case #1: Random walk ($a_1 = 1$)

$$y_t = y_{t-1} + \varepsilon_t$$

$$y_t - y_{t-1} = \varepsilon_t$$

$$\Delta y_t = \varepsilon_t$$

Unit Roots

- In this model, the variance of the error term, ε_t , increases as t increases, in which case OLS will produce a downwardly biased estimate of a_1 (Hurwicz bias).
- Rewrite equation 1 by subtracting y_{t-1} from both sides:

$$y_t - y_{t-1} = a_1 y_{t-1} - y_{t-1} + \varepsilon_t$$

$$\Delta y_t = \delta y_{t-1} + \varepsilon_t \quad (\text{eq. 2})$$

$$\delta = (a_1 - 1)$$

Unit Roots

- $H_0: \delta = 0$ (there is a unit root)
- $H_A: \delta \neq 0$ (there is not a unit root)
- If $\delta = 0$, then we can rewrite equation 2 as

$$\Delta y_t = \varepsilon_t$$

Thus first differences of a random walk time series are stationary, because by assumption, ε_t is purely random.

In general, a time series must be differenced d times to become stationary; it is integrated of order d or $I(d)$. A stationary series is $I(0)$. A random walk series is $I(1)$.

Tests for Unit Roots

- Dickey-Fuller test
 - Estimates a regression using equation 2
 - The usual t-statistic is not valid, thus D-F developed appropriate critical values.
 - You can include a constant, time trend, or both in the test.
 - If you accept the null hypothesis, you conclude that the time series has a unit root.
 - Potential solution: log/first difference the series before proceeding with analysis.

Tests for Unit Roots

- Augmented Dickey-Fuller test (dfuller in STATA)
 - We can use this version if we suspect there is autocorrelation in the residuals.
 - This model is the same as the DF test, but includes lags of the residuals too.
- Phillips-Perron test (pperron in STATA)
 - Makes milder assumptions concerning the error term, allowing for the ε_t to be weakly dependent and heterogeneously distributed.
- Other tests include Variance Ratio test, Modified Rescaled Range test, & KPSS test.
- There are also unit root tests for panel data (Levin et al 2002).

Tests for Unit Roots

- These tests have been criticized for having low power (1-probability(Type II error)).
- They tend to (falsely) accept H_0 too often, finding unit roots frequently, especially with seasonally adjusted data or series with structural breaks. Results are also sensitive to # of lags used in the test.
- Solution involves increasing the frequency of observations or obtaining longer time series.

Example, presidential approval

```
. dfuller presap, lags(1) trend
```

```
Augmented Dickey-Fuller test for unit root          Number of obs   =          317
```

Test Statistic	----- Interpolated Dickey-Fuller -----			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	-4.183	-3.987	-3.427	-3.130

```
-----  
MacKinnon approximate p-value for Z(t) = 0.0047
```

```
. pperron presap
```

```
Phillips-Perron test for unit root          Number of obs   =          318  
                                           Newey-West lags =           5
```

Test Statistic	----- Interpolated Dickey-Fuller -----			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(rho)	-26.181	-20.354	-14.000	-11.200
Z(t)	-3.652	-3.455	-2.877	-2.570

```
-----  
MacKinnon approximate p-value for Z(t) = 0.0048
```

Example, Presidential Approval

- With both tests (ADF, Phillips-Perron), we would reject the null hypothesis of a unit root and conclude that the approval series is stationary.
- This makes sense because it is hard to imagine a bounded variable (0-100) having an infinitely exploding variance over time.
- Yet the series does have some persistence as it trends upward or downward, suggesting that a fractionally integrated model might work best.
- Some studies find that approval has a unit root or near unit root; disagreement about best modeling strategy

Other Types of Integration

- Near Integration
 - Even in cases where $|a_1| < 1$, but close to 1, we still have problems with spurious regression (DeBoef & Granato 1997).
 - Solution: can log or first difference the time series; even though over differencing can induce non-stationarity, short term forecasts are often better
 - DeBoef & Granato also suggest adding more lags of the dependent variable to the model.

Other Types of Integration

- Fractional Integration (Box-Steffensmeier & Smith 1998): $(1-L)^d y_t = \varepsilon_t$

stationary

fractionally
integrated

unit root

$d=0$

$0 < d < 1$

$d=1$

low persistence

high persistence

- Useful for data like presidential approval or interstate conflict/cooperation that have long memored processes, but are not unit roots (especially in the $0.5 < d < 1$ range).

Advanced Topics: Cointegration

- Conventional wisdom is to difference all non-stationary variables used in a regression analysis.
- For example, some scholars pre-whiten time series prior to ARIMA, transfer function, or regression analysis.
- In a multivariate model, however, there may be a linear combination of integrated variables that is stationary.
- Such variables are said to be cointegrated.

Advanced Topics: Cointegration

- This meshes well with theories that expect two or more time series to share an equilibrium relationship.
- For example, some scholars have argued that presidential approval is in equilibrium with economic conditions (Ostrom and Smith 1992).
- If the economy is doing well and approval is too low, it will increase; if the economy is doing poorly, and the president has high approval, it will fall back to the equilibrium level.

Money Demand Model (Enders)

$$m_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + e_t$$

m_t = demand for money

p_t = price level

y_t = real income

r_t = interest rate

e_t = stationary disturbance term

All variables are in logged form.

The model assumes that $\beta_1=1$, $\beta_2>0$, $\beta_3<0$.

Money Demand Model

- Issue: real GDP, the money supply, price level, and the interest rate can all be characterized as nonstationary I(1) variables.
- Yet the theory specifies that there exists a linear combination of these variables that is stationary.

$$e_t = m_t - \beta_0 - \beta_1 p_t - \beta_2 y_t - \beta_3 r_t$$

- Equilibrium theories involving I(1) variables require the existence of a combination of variables that is stationary.
- This is the concept of cointegration as introduced by Engle & Granger (1987).

Advanced Topics: Cointegration

- Variables are in long-run equilibrium when:

$$\beta_1 x_{1t} + \beta_2 x_{2t} + \dots + \beta_n x_{nt} = 0$$

in matrix form; β is the vector $(\beta_1, \beta_2, \dots, \beta_n)$
and x_t is the vector $(x_{1t}, x_{2t}, \dots, x_{nt})'$

$$\beta x_t = 0$$

- The deviation from long run equilibrium, called the equilibrium error, is e_t such that:

$$e_t = \beta x_t$$

Advanced Topics: Cointegration

- The components of the vector $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$ are said to be cointegrated of order (d,b) or $x_t \sim CI(d,b)$ if:
 - All components of x_t are integrated of order d
 - There exists a vector $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ such that the linear combination, $\beta_1 x_{1t} + \beta_2 x_{2t} + \dots + \beta_n x_{nt}$, is integrated of order $(d-b)$ where $b > 0$.
 - The vector β is called the cointegrating vector.
 - In the money demand model, the order is $CI(1,1)$.

Advanced Topics: Cointegration

- Granger (1983) showed that if two variables are cointegrated, then they have an error correction representation (ECM):

$$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_0^* \Delta X_t + \beta_1^* X_{t-1} + \varepsilon_t$$

- In Ostrom and Smith's (1992) model:

$$\Delta A_t = \Delta X_t \beta + \gamma(A_{t-1} - X_{t-1} \nu) + \varepsilon_t$$

where A_t = approval

X_t = quality of life outcome

TABLE 2 Restrictions of the Error Correction Model

Type	ECM Model	Restriction
General	$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_0^* \Delta X_t + \beta_1^* X_{t-1} + \varepsilon_t$	None
Partial Adjustment	$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_0^* X_t + \varepsilon_t$	$\beta_1^* - \beta_0^* = 0$
Static ^a	$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_0^* \Delta X_t + \beta_1^* X_{t-1} + \varepsilon_t$	$\alpha_1^* = -1, \beta_1^* + \alpha_1^* = \beta_0^* - 1$
Finite Distributed Lag ^b	$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_0^* \Delta X_t + \beta_1^* X_{t-1} + \varepsilon_t$	$\alpha_1^* = -1$
Differences ^c	$\Delta Y_t = \alpha_0 + \beta_0^* \Delta X_t + \varepsilon_t$	$\alpha_1^* = 0, \beta_1^* + \alpha_1^* = 0$
Dead Start	$\Delta Y_t = \alpha_0 + \alpha_1^* Y_{t-1} + \beta_1^* X_{t-1} + \varepsilon_t$	$\beta_0^* = 0$
Common Factor ^d	$\Delta Y_t = \alpha_0 + \beta_0^* \Delta X_t + \varepsilon_t, \varepsilon_t = \rho \varepsilon_{t-1} + u_t$	$\rho = 2\alpha_1^* + 1$

^a $k_1 = \beta_0^*$.

^b $k_1 = \beta_1^* + \alpha_1^* + 1$, EC rate 100%.

^cInfinite mean lag length.

^d $\mu = 0$, EC rate 100%, $\rho = \beta_1^* + \alpha_1^*$.

Testing for Cointegration: Step 1

- Suppose two variables are $I(1)$ and we want to see if they are cointegrated.
- Test the variables for their order of integration with unit root tests (DF, ADF, Phillips-Perron, KPSS).
 - If both series are stationary, you can use traditional OLS methods.
 - DeBoef & Keele argue that ECM can be used even for stationary series
 - If they are integrated of different orders, you would need multi-cointegration techniques.

Example: Ostrom and Smith (1992)

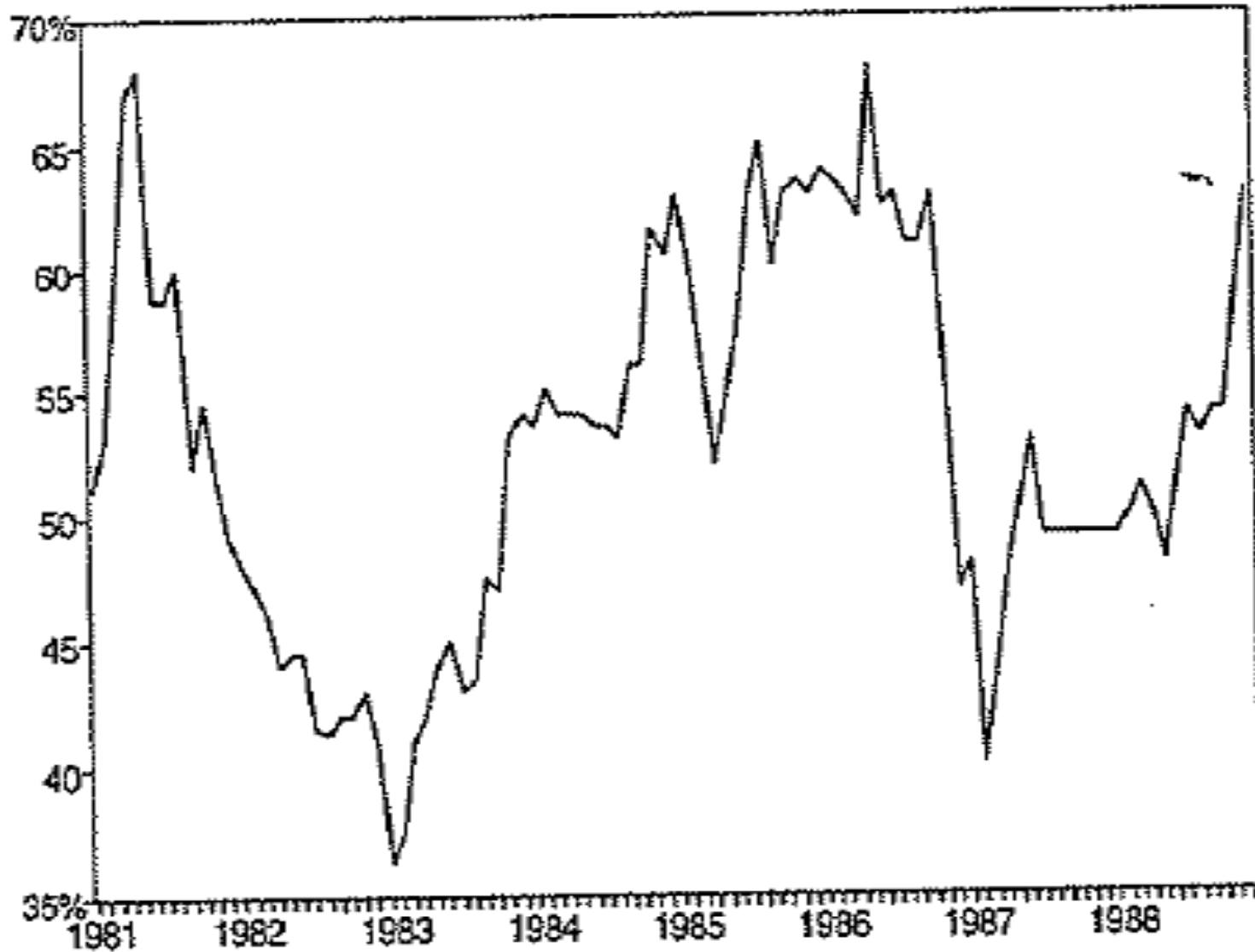


Fig. 1. Presidential approval, January, 1981, through December, 1988

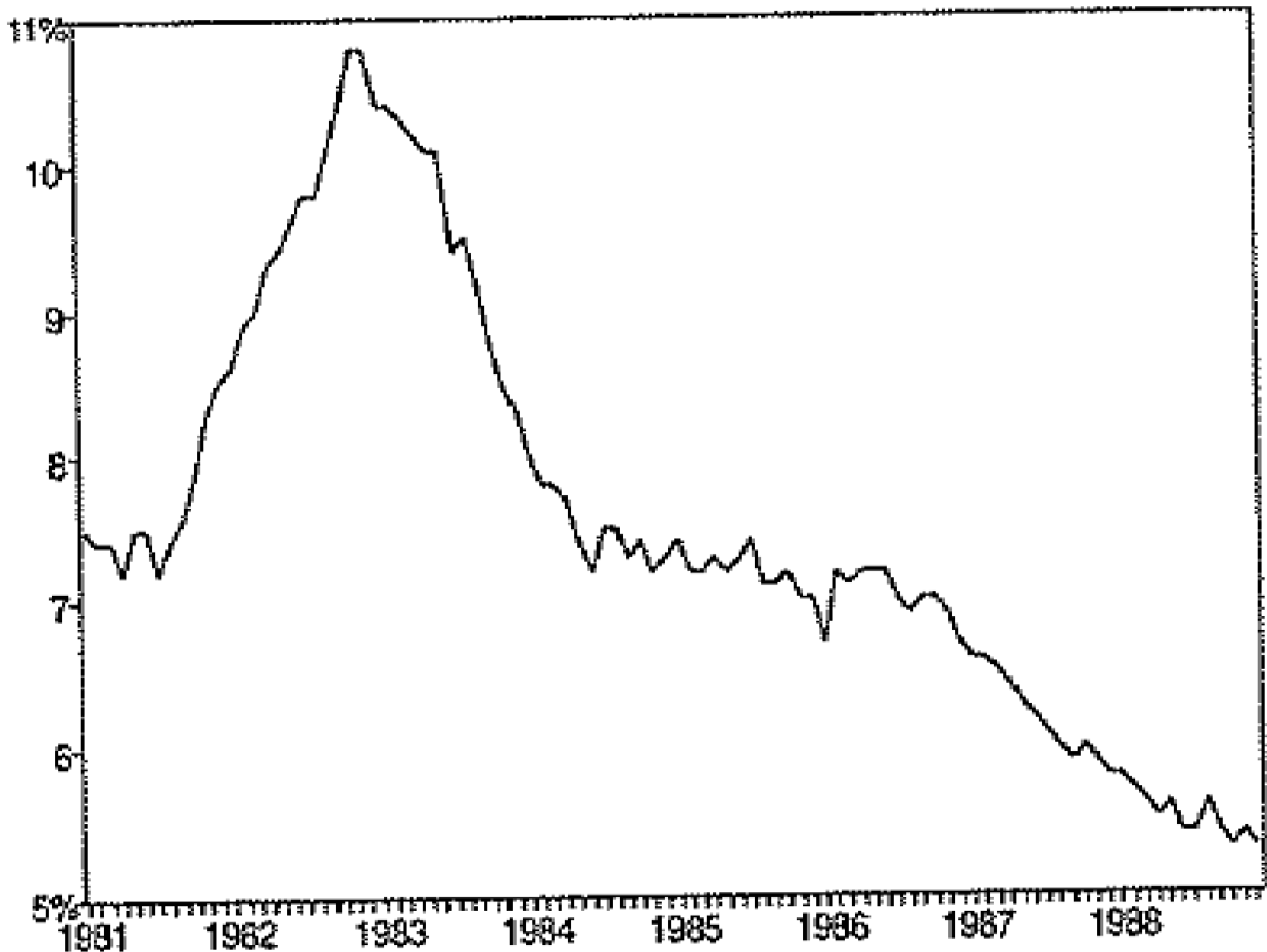


Fig. 2. Unemployment rate, January, 1981, through December, 1988

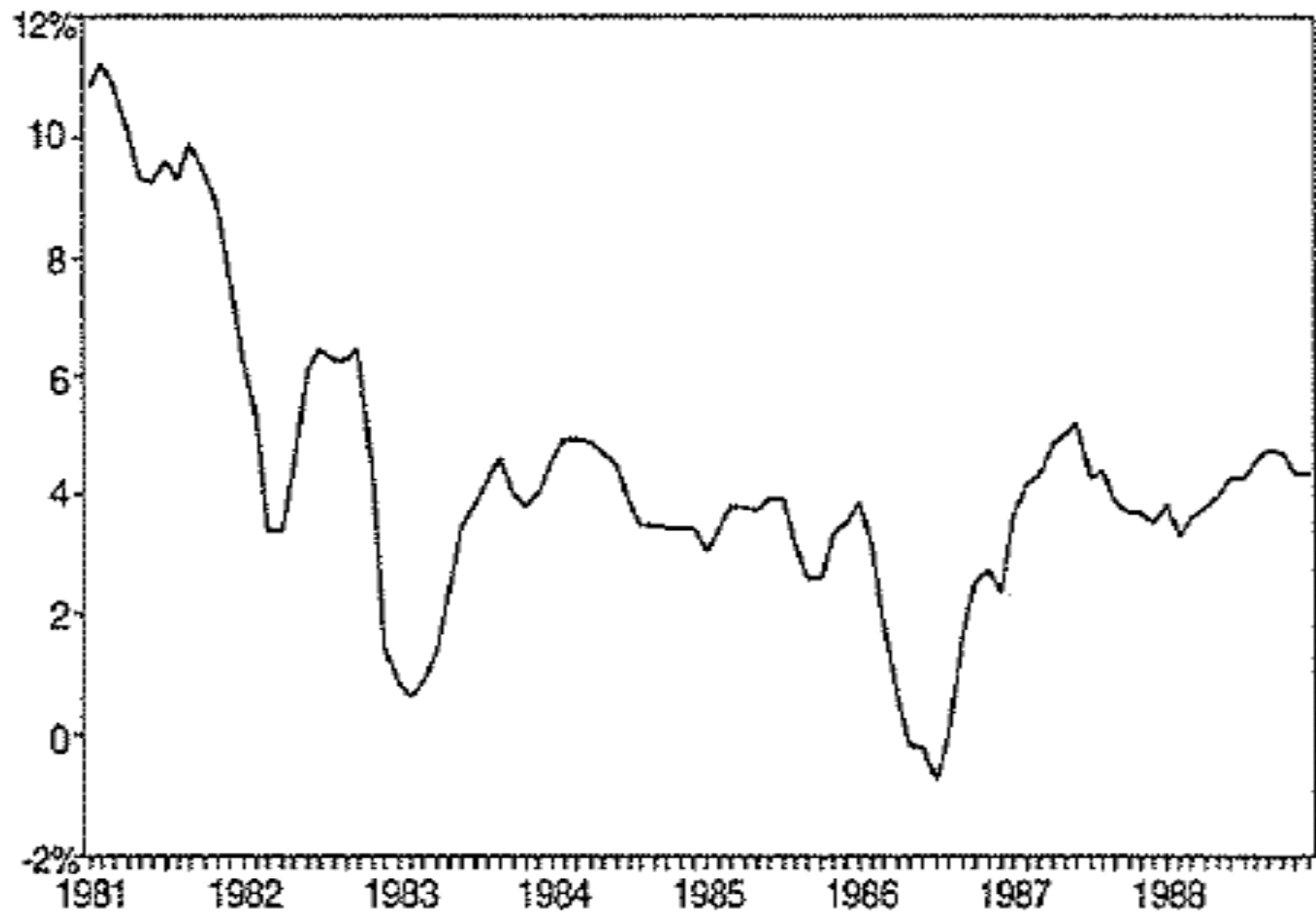


Fig. 3. Annualized inflation rate over six-month spans, January, 1981, through December, 1988

TABLE 2. Dickey-Fuller Tests for Nonstationarity (Reagan Years)

Time-Series	DF ^a	Box-Pierce <i>Q</i>	ADF	Box-Pierce <i>Q</i> ^b
Test for a Unit Root in the Level of Each Series				
Approval	-1.86	8.80	NA	NA
Unemployment	0.25	62.15	-0.36	23.03
Inflation	-2.68	83.45	-2.95	29.73
Test for Trend Stationarity				
Unemployment	-2.09	53.99	-2.51	22.85
Test for a Unit Root in First Differences of Each Series				
Δ Approval	-9.04	8.42	NA	NA
Δ Unemployment	NA	NA	NA	NA
Δ Inflation	-5.41	42.90	-6.33	17.57

Testing for Cointegration: Step 1

- Unit root test suggest inflation is $I(1)$, unemployment is trend stationary, and approval is $I(1)$ or near integrated.
- They also run the unit root tests using data from Truman through Reagan (entire sample), showing similar results.

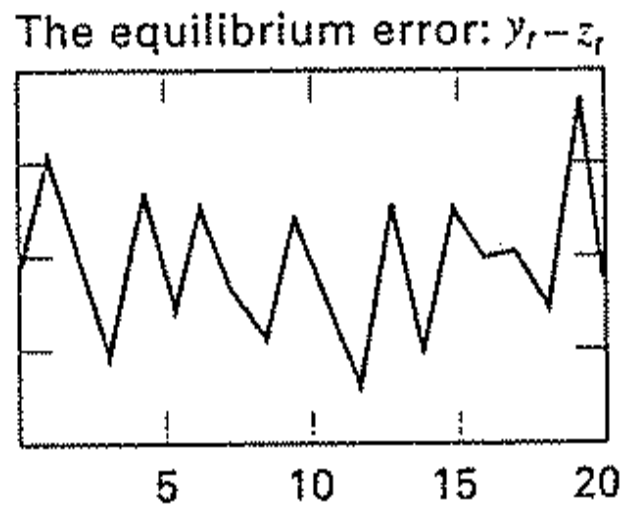
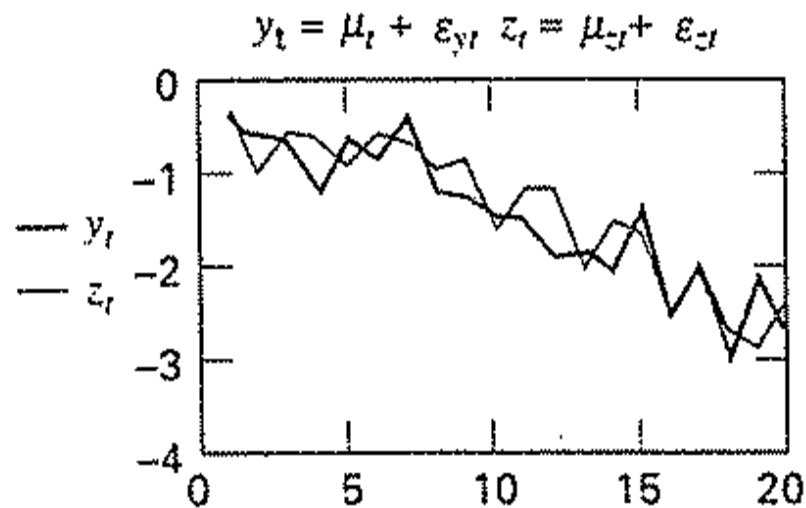
Testing for Cointegration: Step 2

- If two or more series are $I(d)$ where $d > 0$ and d is the same for each series, then estimate the long run equilibrium relationship with OLS.

$$y_t = \beta_0 + \beta_1 z_t + e_t$$

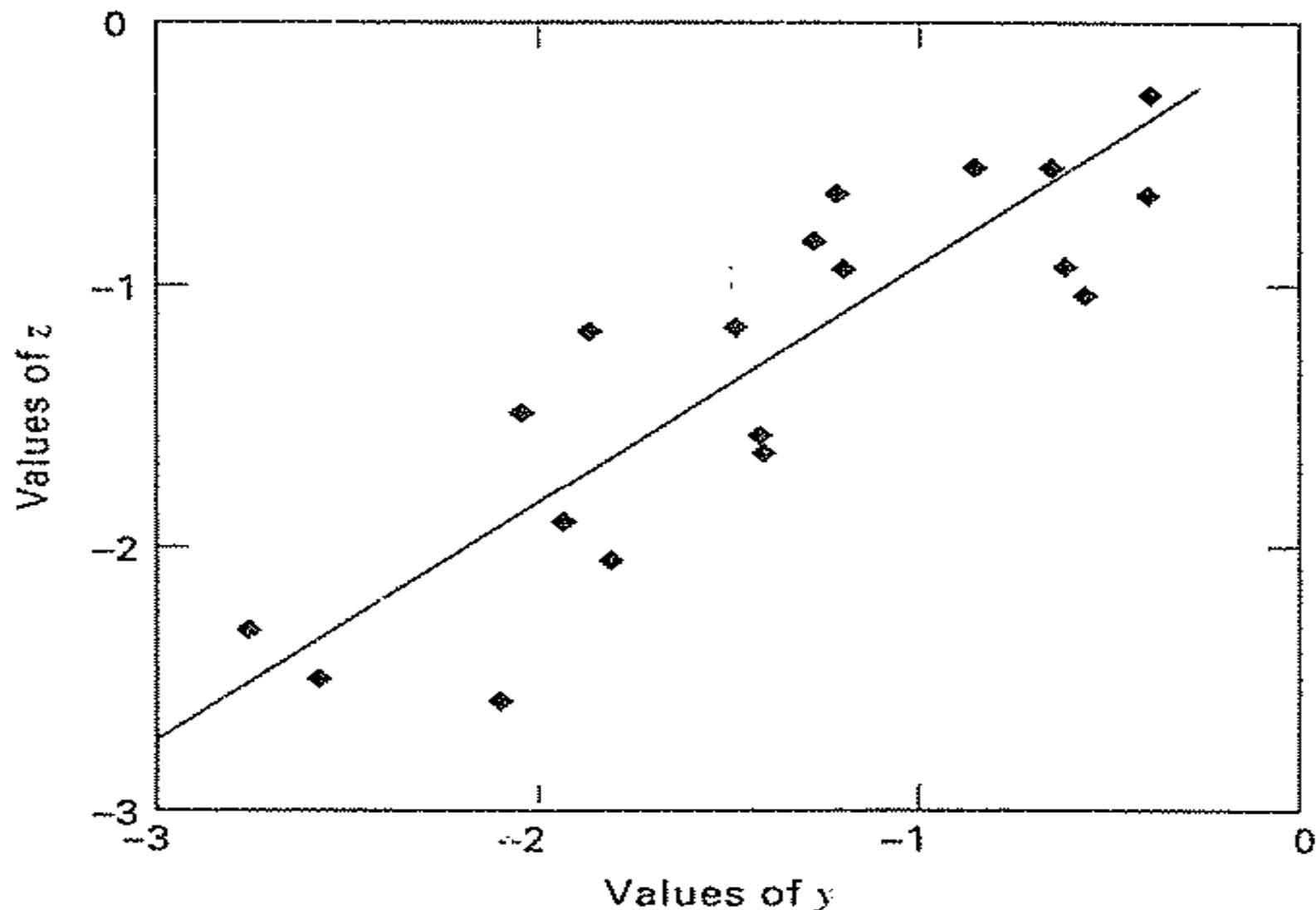
- Save the residuals.
- See if they are stationary using the same unit root tests as in step #1.
 - Should see a high R^2

CASE 1 The $\{y_t\}$ and $\{z_t\}$ sequences are both random walk plus noise processes. Although each is nonstationary, the two sequences have the same stochastic trend; hence, they are cointegrated such that the linear combination $(y_t - z_t)$ is stationary. The equilibrium error term is an $I(0)$ process.



Both series are $I(1)$, but $y_t - z_t$ is stationary.

Figure 6.1 Scatter plot of cointegrated variables.



The scatter plot was drawn using the {y} and {z} sequences from Case 1 of Worksheet 6.1. Since both series decline over time, there appears to be a positive relationship between the two. The equilibrium regression line is shown.

Testing for Cointegration: Step 2

- We are testing $\Delta e_t = a_1 e_{t-1} + \varepsilon_t$
 - $H_0: a_1 = 0$ (unit root)
 - $H_A: a_1 \neq 0$ (no unit root)
- If you accept the null hypothesis (find a unit root in the residuals), you would conclude the two series are NOT cointegrated.
- If you reject the null hypothesis of a unit root, you would conclude the two series are cointegrated.
- Issue: standard DF tests fail, so you should use the Engle & Granger critical values or Johansen tests.

Testing for Cointegration: Step 2

- Ostrom & Smith estimate each variable as the DV (three models); Table 5.
- The Dickey Fuller tests allow for rejection of a unit root for the approval and unemployment cointegrating regressions.
- Note R^2 is high for approval and unemployment regressions, but not inflation.

TABLE 5. Cointegrating Regressions and Tests for Cointegration

	Endogenous Variable		
	Approval	Unemployment	Inflation
Constant	105.09 (37.79) ^a	15.35 (38.75)	24.81 (3.53)
Approval	—	-0.13 (-18.75)	-0.20 (-3.01)
Unemployment	-6.11 (-18.75)	—	-1.23 (-2.70)
Inflation	-0.46 (-3.01)	-0.06 (-2.70)	—
Assassination attempt	7.29 (3.81)	0.78 (2.70)	6.36 (5.36)
Iran-Contra scandal	-16.49 (-15.44)	-2.61 (-22.98)	-3.43 (-2.65)
Goodness of fit			
<i>R</i> ²	.82	.89	.27
SE	3.32	0.48	2.19
DW	1.08	0.94	0.38
Residual test for cointegration			
DF test ^b	-5.72	-5.30	-3.70
ADF test	NA	NA	-5.31

^aThe quantities in parentheses are the *t*-ratios produced by the regression package. However, because of the nonstationarity of some of the regressors, these coefficients may have nonstandard distributions, which makes drawing inferences from individual coefficients problematic (Engle and Granger 1987).

^bCritical values for the DF and ADF tests for cointegration (MacKinnon 1991) are: $\alpha = .01$, -5.20; $\alpha = .05$, -4.57; $\alpha = .10$, -4.25.

Testing for Cointegration: Step 3

- Estimate an unrestricted VAR to help determine best model.
- Estimate the error correction model (ECM) using the lagged residuals from step 2 as instruments for the long run equilibrium term.
- Then estimate the model using OLS.
 - Can be single equation if one variable is weakly exogenous or multi-equation (VAR) if they are endogenously related.

TABLE 6. Unrestricted VAR Results

	Endogenous Variable					
	Δ Inflation		Δ Unemployment		Δ Approval	
	Coefficient	<i>t</i> -ratio ^a	Coefficient	<i>t</i> -ratio ^a	Coefficient	<i>t</i> -ratio ^a
Δ Inflation (-1)	0.26	2.37	-0.01	-0.16	0.20	0.38
Δ Inflation (-2)	-0.05	-0.45	-0.02	-0.49	-0.29	-0.52
Δ Inflation (-3)	-0.08	-0.72	-0.03	-0.78	0.18	0.32
Δ Inflation (-4)	-0.04	-0.33	0.02	0.47	0.99	1.77
Δ Inflation (-5)	-0.11	-1.01	0.03	0.84	0.05	0.09
Δ Inflation (-6)	-0.40	-3.96	-0.07	-2.25	0.20	0.40
Δ Unemployment (-1)	-0.72	-1.94	-0.21	-1.90	3.31	1.80
Δ Unemployment (-2)	-0.76	-2.12	-0.02	-0.15	-2.65	-1.49
Δ Unemployment (-3)	-0.37	-1.03	0.15	1.33	0.39	0.22
Δ Unemployment (-4)	-0.12	-0.34	0.19	1.85	-0.70	-0.41
Δ Unemployment (-5)	-0.32	-0.91	0.20	1.89	1.36	0.79
Δ Unemployment (-6)	0.21	0.59	0.31	2.86	0.58	0.33
Δ Approval (-1)	-0.00	-0.18	-0.00	-0.25	0.23	2.01
Δ Approval (-2)	-0.04	-1.85	-0.00	-0.31	0.28	2.48
Δ Approval (-3)	-0.03	-1.32	-0.01	-1.61	0.19	1.90
Δ Approval (-4)	-0.00	-0.00	-0.00	-0.12	-0.21	-2.10
Δ Approval (-5)	0.01	0.27	-0.01	-1.15	-0.01	-0.06
Δ Approval (-6)	-0.00	-0.03	0.00	0.39	0.05	0.46
Assassination attempt (-1)	-0.34	-0.48	-0.11	-0.50	9.56	2.72
Iran-Contra scandal (-1)	0.20	0.45	-0.18	-1.33	-9.89	-4.58
Approval (-1)	-0.00	-0.16	-0.00	-0.12	-0.62	-5.24
Unemployment (-1)	-0.01	-0.06	-0.07	-1.55	-3.67	-4.78
Inflation (-1)	-0.06	-1.36	0.05	3.89	-0.57	-2.75
Constant	0.29	0.11	0.44	0.56	65.17	5.11
<i>R</i> ²	.63		.55		.46	
SE	.51		.16		2.54	
DW	1.84		2.11		1.77	
Box-Pierce <i>Q</i> (20 <i>df</i>)	18.35		17.59		10.66	

TABLE 7. Bloc *F*-Tests

	Endogenous Variable					
	Δ Inflation		Δ Unemployment		Δ Approval	
	<i>F</i>	<i>p</i> -value	<i>F</i>	<i>p</i> -value	<i>F</i>	<i>p</i> -value
Δ Inflation	9.57	.00	1.03	.41	0.91	.49
Δ Unemployment	1.92	.09	3.24	.01	1.29	.27
Δ Approval	0.84	.54	0.70	.65	3.05	.01

Testing for Cointegration, Step 3

- The Granger Causality tests show that neither differenced inflation nor differenced unemployment granger causes approval.
- Inflation and unemployment influence changes in approval only by creating some disequilibrium in rewards and punishments that gets corrected over time.
- We can treat inflation and unemployment as weakly exogenous, and thus estimate a single equation model.

TABLE 8. OLS Estimates of Approval Equilibrium Model

	Endogenous Variable		
	Δ Approval		
	Coefficient	<i>t</i> -ratio	<i>p</i> -value
Δ Approval (-1)	0.18	2.06	.04 ^a
Δ Approval (-2)	0.20	2.15	.03 ^a
Δ Approval (-3)	0.15	1.78	.08 ^a
Δ Approval (-4)	-0.15	-1.90	.06 ^a
Δ Approval (-5)	-0.07	-0.85	.40 ^a
Δ Approval (-6)	-0.04	-0.52	.60 ^a
Enhancing—domestic events	3.40	2.52	.01
Enhancing—international events	1.53	2.30	.01
Enhancing—personal events	2.37	1.69	.05
Diminishing—domestic events	-0.88	-0.98	.16
Diminishing—international events	-2.52	-1.55	.06
Diminishing—personal events	-2.71	-1.60	.06
Speeches—domestic	0.86	1.00	.16
Speeches—international	-0.26	-0.36	.84
Foreign travel	1.43	2.14	.02
Equilibrium error, Z (-1)	-0.56	-6.29	.00
Constant	-0.53	-1.56	.12 ^a

Testing for Cointegration, Step 3

- The equilibrium error ($Z(-1)$) is statistically significant. The value of -0.56 indicates that 56% of the previous month's disequilibrium is corrected in the next period and $(.56)^L$ in subsequent periods.
- We also see that lags of approval are significant as a block suggesting a type of momentum generating effect from media announcements of Reagan's approval ratings.
- Dramatic and ordinary events (except speeches) also influence the president's approval rating.

Testing for Cointegration: Step 4

- Assess model adequacy
- If residuals are serially correlated, lag lengths may be too short.
- Look at significance and size of speed of adjustment parameters.
- You can plot the impulse responses, which should converge to zero, because the cointegrating relationship is $I(0)$.

Testing for Cointegration, Step 5

- Step 5: Assess model adequacy
- Tests show white noise residuals with no remaining autocorrelation or heteroskedasticity

Goodness-of-fit measures	
R^2 (changes)	0.54
Adjusted R^2 (changes)	0.44
R^2 (levels)	0.93
Adjusted R^2 (levels)	0.91
Log likelihood	-187.84
SE	2.22
Percent correct turning points	0.77
Residual diagnostics	
DW	1.82
ARCH χ^2 (6)	6.17
Jarque-Bera Normality χ^2 (2)	1.43
Box-Pierce Q χ^2 (20)	11.50

Plot Impulse Response Function

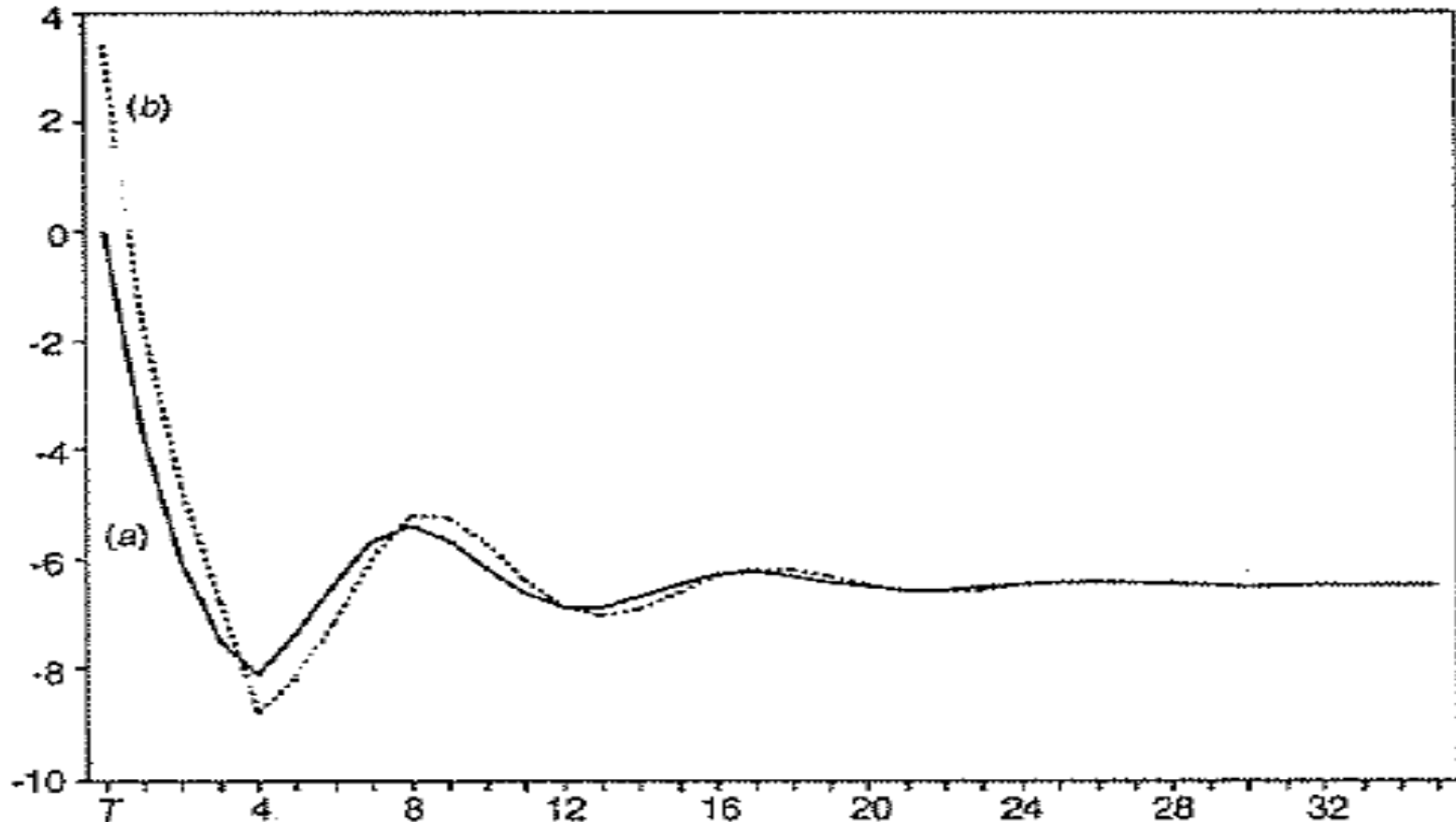


Fig. 4. Response of approval to (a) a sustained one-point increase in unemployment at T and (b) the joint occurrence of the unemployment increase and an approval-enhancing event

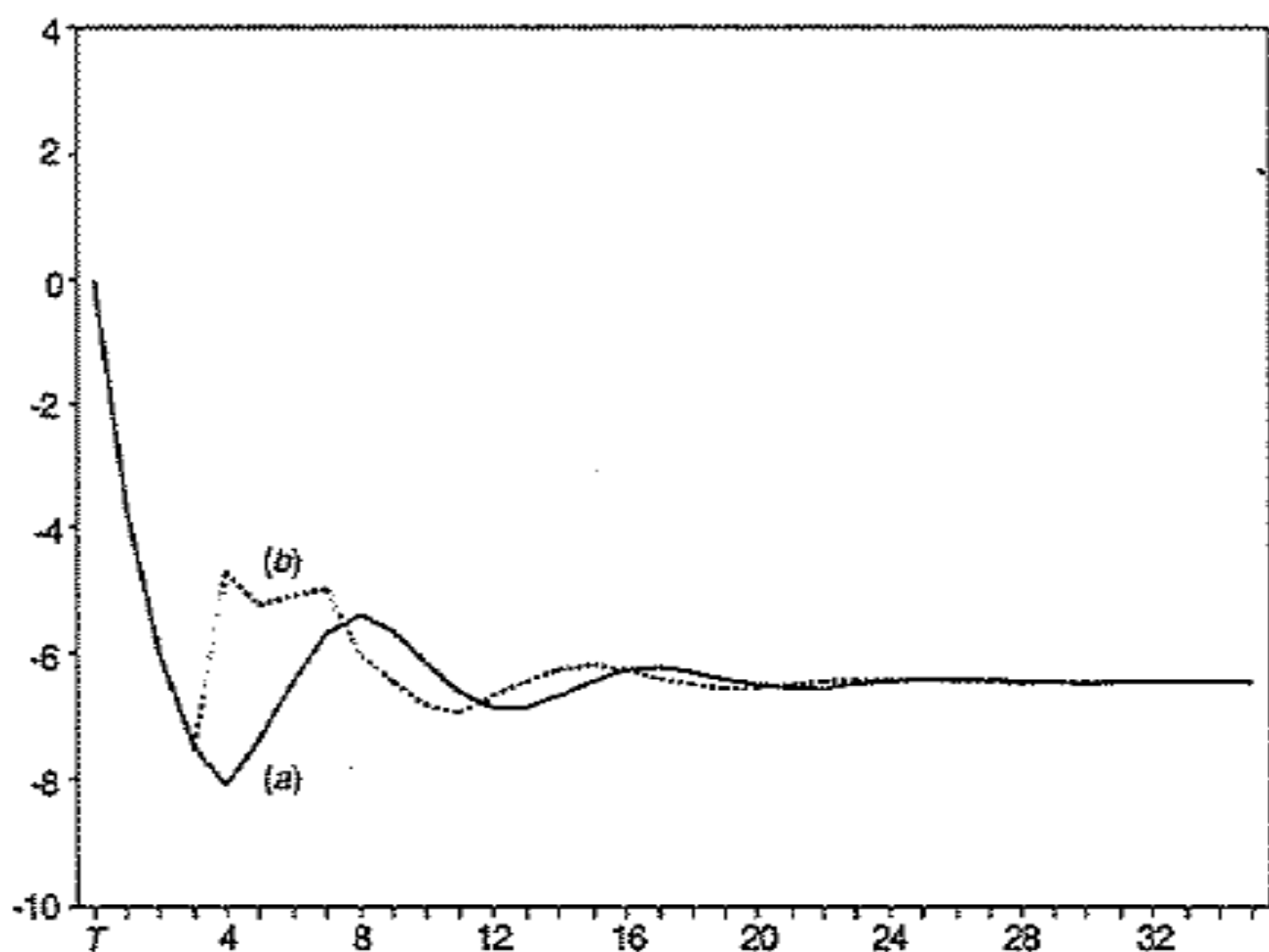


Fig. 5. Response of approval to (a) a sustained one-point increase in unemployment at T and (b) the unemployment increase at T and an approval-enhancing event at $T + 4$

Conclusion

- A president might optimally manage levels of public support through the use of a mixed strategy, relying on the influence of approval-enhancing events and economic policy manipulation.
- Modeling time series requires careful analysis of dynamic specification issues.
- We must take into account dynamics of the DV when estimating the size of the effects of our IVs.