Example 3: Time-Invariant Predictors of Practice Effects (uses same data as Example 2)

In this example we will examine time-invariant predictors of individual differences in intercepts, linear slopes, and quadratic slopes representing improvement in RT (in msec) across six practice sessions. We will examine age, abstract reasoning, and education in sequential conditional (predictor) models.

SAS Code for Data Manipulation:

```
* Centering level-2 predictor variables for analysis;
DATA work.example23; SET work.example23;
       age80  = age - 80;         * Convenient value;
       reas22 = absreas - 22;     * Near sample mean;
       LABEL age80  = "age80: Age Centered (0=80)"
             reas22 = "reas22: Abstract Reasoning Centered (0=22)";
       * Make education a grouping variable for purpose of demonstration only;
            IF educyrs LE 12                   THEN educgrp=1;
       ELSE IF educyrs GT 12 AND EducYrs LE 16 THEN educgrp=2;
       ELSE IF educyrs GT 16                   THEN educgrp=3;
       ELSE IF educyrs = .                     THEN educGrp=.;
       LABEL educgrp = "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)";

* Removing cases with missing predictors;
       IF NMISS(age80, reas22, educgrp)>0 THEN DELETE;
RUN;
```

SPSS Code for Data Manipulation:

```
* Centering level-2 predictor variables for analysis.
DATASET ACTIVATE example23 WINDOW=FRONT.
COMPUTE age80  = age - 80.
COMPUTE reas22 = absreas - 22.
VARIABLE LABELS
       age80 "age80: Age Centered (0=80)"
       reas22 "reas22: Abstract Reasoning Centered (0=22)".
* Make education a grouping variable for purpose of demonstration only.
IF educyrs LE 12                   educgrp=1.
IF educyrs GT 12 AND educyrs LE 16 educgrp=2.
IF educyrs GT 16                   educgrp=3.
VARIABLE LABELS educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)".

* Removing cases with missing predictors.
SELECT IF (NVALID(age80, reas22, educgrp)=3).
EXECUTE.
```

STATA Code for Data Manipulation:

```
* centering level-2 predictor variables for analysis
gen age80 = age - 80
gen reas22 = absreas - 22
label variable age80 "age80: Age Centered (0=80 years)"
label variable reas22 "reas22: Abstract Reasoning Centered (0=22)"
* make education a grouping variable for purpose of demonstration only
gen educgrp=.
replace educgrp=1 if (educyrs <= 12)
replace educgrp=2 if (educyrs > 12 & educyrs <= 16)
replace educgrp=3 if (educyrs > 16)
label variable educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)"

 * create new variable to hold number of missing cases
 * then drop cases with incomplete predictors
egen nummiss = rowmiss(age80 reas22 educgrp)
drop if nummiss>0
```

Model 3b. Random Quadratic Time Baseline (in ML now)

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + U_{2i}$

```
TITLE1 "SAS Model 3b: Random Quadratic Time Baseline in ML";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite OUTPM=work.TimePred;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;
PROC CORR NOSIMPLE DATA=work.TimePred; VAR nm3rt pred; RUN;

TITLE "SPSS Model 3b: Random Quadratic Time Baseline in ML".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predtime).
CORRELATIONS nm3rt predtime.

* STATA Model 3b: Random Quadratic Time Baseline in ML
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess,  || id: c1sess c1sess2,  ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store Baseline,  // save LL for LRT
      predict predtime           // save fixed-effect predicted outcomes
corr nm3rt predtime              // get total r to make r2
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =       606
Group variable: id                              Number of groups   =       101
                                                Obs per group: min =         6
                                                               avg =       6.0
                                                               max =         6
                                                Wald chi2(2)       =     72.45
Log likelihood = -4160.8833                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------------
            nm3rt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
------------------+-----------------------------------------------------------------
           c1sess |  -120.8999   19.94803    -6.06   0.000    -159.9973   -81.80251
c.c1sess#c.c1sess |   13.86561   3.398459     4.08   0.000     7.204756    20.52647
            _cons |    1945.85   53.58259    36.31   0.000      1840.83     2050.87
------------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(c1sess) |   25437.86   5781.419      16293.81    39713.52
                var(c1sess2) |      622.8     169.99      364.7687    1063.358
                  var(_cons) |   273306.9   40831.76      203930.4    366285.1
          cov(c1sess,c1sess2) |  -3837.723   968.8047     -5736.545     -1938.9
            cov(c1sess,_cons) |  -35261.67    11771.5     -58333.38   -12189.95
           cov(c1sess2,_cons) |   3845.378   1921.468      79.37031    7611.386
-----------------------------+------------------------------------------------
```

```
              var(Residual) |    20298.2   1649.119       17310.2    23801.98
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(6) =    891.99   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.

.         estat ic, n(101),
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    101           .   -4160.883     10     8341.767    8367.918
-----------------------------------------------------------------------------
               Note:  N=101 used in calculating BIC
```

In ML, the #parms is ALL parms (both sides of model). So STATA's versions should agree with other programs.

```
             |    nm3rt predtime
-------------+------------------
       nm3rt |   1.0000
    predtime |   0.1917   1.0000
```

R =.1917, so R^2 for time = .0367

The model for the means (fixed linear and quadratic session effects so far) accounted for ~4% of the variance in RT.

Model 4a. Age as Predictor of Intercept, Linear, and Quadratic Time Slopes

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$$

Level 2:

$$\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$$

$$\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$$

$$\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$$

```
TITLE1 "SAS Model 4a: Age as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                          / SOLUTION DDFM=Satterthwaite OUTPM=work.AgePred;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
      REPEATED session / TYPE=VC SUBJECT=ID;
      * Requesting additional effects for age;
      ESTIMATE "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0;
      ESTIMATE "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1;
      ESTIMATE "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4;
      ESTIMATE "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9;
      ESTIMATE "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16;
      ESTIMATE "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25;
RUN; PROC CORR NOSIMPLE DATA=work.AgePred; VAR nm3rt pred; RUN;
```

```
TITLE "SPSS Model 4a: Age as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predage)
      /TEST = "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0
      /TEST = "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1
      /TEST = "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4
      /TEST = "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9
      /TEST = "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16
      /TEST = "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25.
CORRELATIONS nm3rt predage.
```

```
* STATA Model 4a: Age as Predictor of Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                        ///
      c.age80 c.age80#c.c1sess c.age80#c.c1sess#c.c1sess,  ///
      || id: c1sess c1sess2,                                ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store age,            // save LL for LRT
      lrtest Age Baseline,            // LRT against non-age baseline
      predict predage                 // save fixed-effect predicted outcomes
      margins, at(c.c1sess=(0(1)5)) dydx(c.age80) vsquish     // age slope per session
      margins, at(c.c1sess=(0(1)5) c.age80=(-5 0 5)) vsquish  // predictions per session
      marginsplot, name(predicted_age, replace)                // plot age predictions
corr nm3rt predage         // get total r to make r2
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =       606
Group variable: id                              Number of groups   =       101
                                                Obs per group: min =         6
                                                               avg =       6.0
                                                               max =         6
                                                Wald chi2(5)       =     88.55
Log likelihood = -4155.1009                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------------------
                   nm3rt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------------
                  c1sess |  -121.8325   19.66948    -6.19   0.000    -160.3839   -83.28099
       c.c1sess#c.c1sess |   13.97744   3.375686     4.14   0.000     7.361221    20.59367
                   age80 |   29.04954   8.377364     3.47   0.001     12.63021    45.46887
        c.age80#c.c1sess |  -5.594634   3.251901    -1.72   0.085    -11.96824    .7789759
c.age80#c.c1sess#c.c1sess |   .6709122    .558093     1.20   0.229     -.42293    1.764754
                   _cons |   1950.692   50.67139    38.50   0.000     1851.378    2050.006
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(c1sess) |   24293.61   5623.947      15432.62    38242.33 → linear var down by 4.50%
                var(c1sess2) |   606.3449   167.7546      352.5508     1042.84 → quad var down by 2.64%
                  var(_cons) |   242456.1   36492.45      180516.8    325648.3 → intercept var down 11.29%
         cov(c1sess,c1sess2) |  -3700.505    949.404     -5561.302   -1839.707
           cov(c1sess,_cons) |  -29320.18   10868.45     -50621.95   -8018.411
          cov(c1sess2,_cons) |   3132.873   1793.883     -383.0738    6648.819
-----------------------------+------------------------------------------------
               var(Residual) |    20298.2   1649.119       17310.2    23801.98 → residual var not reduced
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(6) =   857.76   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.

.         estat ic, n(101),
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    101           .   -4155.101     13     8336.202    8370.198
-----------------------------------------------------------------------------
               Note:  N=101 used in calculating BIC

.         estimates store Age,           // save LL for LRT
.         lrtest Age Baseline,           // LRT against non-age baseline

Likelihood-ratio test                            LR chi2(3)  =     11.56
(Assumption: Baseline nested in Age)             Prob > chi2 =    0.0090
```

Is the age model (4a) better than the baseline random quadratic model (3b)?

Yes, $-2\Delta LL=11.6$ on df=3, p=.009

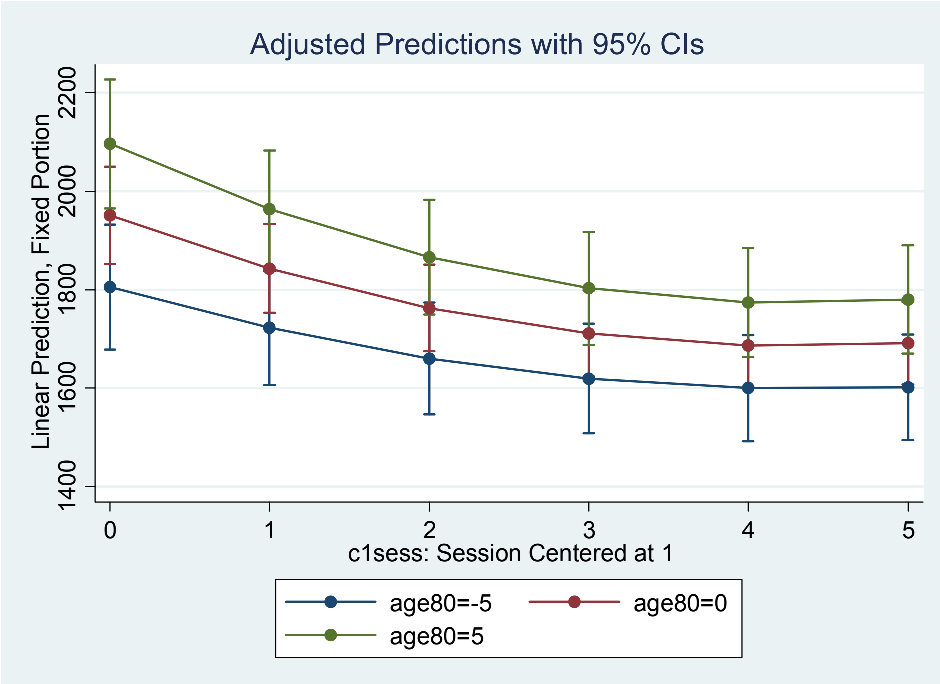
```
.         predict predage                          // save fixed-effect predicted outcomes
(option xb assumed)
.         margins, at(c1sess=(0(1)5)) dydx(age80) vsquish     // age slope per session

Average marginal effects                           Number of obs   =        606

Expression   : Linear prediction, fixed portion, predict()
dy/dx w.r.t. : age80
1._at        : c1sess          =           0
2._at        : c1sess          =           1
3._at        : c1sess          =           2
4._at        : c1sess          =           3
5._at        : c1sess          =           4
6._at        : c1sess          =           5
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
age80    _at |
          1  |   29.04954   8.377364     3.47   0.001     12.63021    45.46887
          2  |   24.12582   7.609705     3.17   0.002     9.211068    39.04056
          3  |   20.54392   7.459286     2.75   0.006     5.923987    35.16385
          4  |   18.30385   7.330177     2.50   0.013     3.936962    32.67073
          5  |    17.4056   7.071475     2.46   0.014     3.545761    31.26543
          6  |   17.84917   7.054461     2.53   0.011     4.022683    31.67566
------------------------------------------------------------------------------
```

These are the simple slopes for age at each session.

```
.         margins, at(c1sess=(0(1)5) age80=(-5 0 5)) vsquish  // predictions per session

Adjusted predictions                               Number of obs   =        606
Expression   : Linear prediction, fixed portion, predict()
1._at        : c1sess          =           0
               age80           =          -5
2._at        : c1sess          =           0
               age80           =           0
3._at        : c1sess          =           0
               age80           =           5
(output continues for all other sessions)
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   1805.444   64.84687    27.84   0.000     1678.347    1932.542
          2  |   1950.692   50.67139    38.50   0.000     1851.378    2050.006
          3  |    2095.94   66.62638    31.46   0.000     1965.354    2226.525
          4  |   1722.208   58.90463    29.24   0.000     1606.757    1837.659
          5  |   1842.837   46.02812    40.04   0.000     1752.623     1933.05
          6  |   1963.466   60.52108    32.44   0.000     1844.847    2082.085
          7  |   1660.217   57.74028    28.75   0.000     1547.048    1773.386
          8  |   1762.937    45.1183    39.07   0.000     1674.506    1851.367
          9  |   1865.656   59.32478    31.45   0.000     1749.382    1981.931
         10  |   1619.472   56.74089    28.54   0.000     1508.262    1730.682
         11  |   1710.991   44.33737    38.59   0.000     1624.092    1797.891
         12  |   1802.511   58.29796    30.92   0.000     1688.249    1916.773
         13  |   1599.973   54.73834    29.23   0.000     1492.688    1707.258
         14  |   1687.001   42.77258    39.44   0.000     1603.168    1770.834
         15  |   1774.029   56.24045    31.54   0.000       1663.8    1884.258
         16  |    1601.72   54.60664    29.33   0.000     1494.693    1708.747
         17  |   1690.966   42.66967    39.63   0.000     1607.335    1774.597
         18  |   1780.212   56.10514    31.73   0.000     1670.247    1890.176
------------------------------------------------------------------------------
```

The pattern of the interaction is shown by the simple effects of age at each session, graphed below.

```
.          marginsplot, name(predicted_age, replace)                   // plot age predictions
```




```
  Variables that uniquely identify margins: c1sess age80

. corr nm3rt predage      // get total r to make r2
(obs=606)

             |    nm3rt  predage
-------------+------------------
       nm3rt |   1.0000
     predage |   0.3269   1.0000
```

R =.3269, so R^2 for time+age = .1069

The fixed effects of time before accounted for ~3.7% of the variance in RT, so there is a net increase of ~7% due to age.

Model 5a. +Abstract Reasoning as Predictor of Intercept, Linear, and Quadratic Time Slopes

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$$

Level 2:

$$\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + U_{0i}$$

$$\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + U_{1i}$$

$$\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reason}_i - 22\right) + U_{2i}$$

```
TITLE1 "SAS Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS ID session;
       MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                     reas22 c1sess*reas22 c1sess*c1sess*reas22
                           / SOLUTION DDFM=Satterthwaite OUTPM=work.ReasPred;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
       REPEATED session / TYPE=VC SUBJECT=ID;
RUN; PROC CORR NOSIMPLE DATA=work.ReasPred; VAR nm3rt pred; RUN;
```

```
TITLE "SPSS Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22 c1sess*c1sess*reas22
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predreas).
CORRELATIONS nm3rt predreas.

* STATA Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                              ///
      c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess             ///
      c.reas22 c.reas22#c.c1sess c.reas22#c.c1sess#c.c1sess,          ///
      || id: c1sess c1sess2,                                          ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store Reas,          // save LL for LRT
      lrtest Reas Age,               // LRT against age baseline
      predict predreas               // save fixed-effect predicted outcomes
corr nm3rt predreas                  // get total r to make r2
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =       606
Group variable: id                              Number of groups   =       101
                                                Obs per group: min =         6
                                                               avg =       6.0
                                                               max =         6
                                                Wald chi2(8)       =    103.88
Log likelihood = -4148.8645                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------------
                  nm3rt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------
                 c1sess |  -119.7417   19.77414    -6.06   0.000    -158.4983   -80.98505
       c.c1sess#c.c1sess |   13.30362    3.36557     3.95   0.000     6.707229    19.90002
                  age80 |   22.27817   8.601751     2.59   0.010     5.419047    39.13729
        c.age80#c.c1sess |  -6.492074   3.424732    -1.90   0.058    -13.20443    .2202772
c.age80#c.c1sess#c.c1sess |   .9601368   .5828914     1.65   0.100    -.1823093    2.102583
                 reas22 |  -27.10041   11.11411    -2.44   0.015    -48.88366   -5.317155
       c.reas22#c.c1sess |  -3.591742   4.425011    -0.81   0.417     -12.2646    5.081121
c.reas22#c.c1sess#c.c1sess |   1.157537   .7531395     1.54   0.124    -.3185897    2.633663
                  _cons |   1966.467   49.66585    39.59   0.000     1869.124    2063.811
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
               var(c1sess)   |   24040.63    5589.24      15242.24    37917.78 → linear var down by 1.04%
              var(c1sess2)   |   580.0652   164.1907      333.0729    1010.216 → quad var down by 4.33%
                var(_cons)   |   228049.3   34467.25      169581.6    306675.4 → intercept var down by 5.94%
       cov(c1sess,c1sess2)   |  -3618.966   937.0759     -5455.601   -1782.331
         cov(c1sess,_cons)   |  -31229.47   10655.91     -52114.67   -10344.27
        cov(c1sess2,_cons)   |   3748.206   1747.738      322.7024    7173.709
-----------------------------+------------------------------------------------
             var(Residual)   |   20298.18   1649.113      17310.18    23801.94 → residual var not reduced
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(6) =   832.43   Prob > chi2 = 0.0000
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    101           .   -4148.864     16     8329.729    8371.571
-----------------------------------------------------------------------------
```

```
.         lrtest Reas Age,                      // LRT against age baseline

Likelihood-ratio test                                  LR chi2(3)   =     12.47
(Assumption: Age nested in Reas)                       Prob > chi2 =    0.0059

. corr nm3rt predreas                                  // get total r to make r2
(obs=606)
             |    nm3rt predreas
-------------+------------------
       nm3rt |   1.0000
    predreas |   0.4011   1.0000
```

Is the reasoning model (5a) better than the age model (4a)?

Yes, −2ΔLL = 12.5 on df=3, *p* =.0059, so ΔR^2 is significant

R =.4011, so R^2 for time+age+reas = .1609

The fixed effects of time and age before accounted for ~10.7% of the variance in RT, so there is a net increase of ~5.4% due to reasoning.

Model 5b. Abstract Reasoning on Intercept and Linear Time Slope Only

$$
\begin{aligned}
&\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti} \\
&\text{Level 2:} \\
&\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + U_{0i} \\
&\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + U_{1i} \\
&\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}
\end{aligned}
$$

```
TITLE1 "SAS Model 5b: Reasoning on Intercept and Linear Time Slope Only";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                    reas22 c1sess*reas22
                  / SOLUTION DDFM=Satterthwaite OUTPM=work.ReasPred2;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
      REPEATED session / TYPE=VC SUBJECT=ID;
      * Requesting additional effects for reasoning instead;
      ESTIMATE "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0;
      ESTIMATE "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1;
      ESTIMATE "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2;
      ESTIMATE "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3;
      ESTIMATE "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4;
      ESTIMATE "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5;
RUN; PROC CORR NOSIMPLE DATA=work.ReasPred2; VAR nm3rt pred; RUN;

TITLE "SPSS Model 5b: +Reasoning as Predictor of Intercept, Linear Time Slope Only".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predreas2)
      /TEST = "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0
      /TEST = "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1
      /TEST = "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2
      /TEST = "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3
      /TEST = "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4
      /TEST = "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5.
CORRELATIONS nm3rt predreas2.

 * STATA Model 5b: +Reasoning as Predictor of Intercept, Linear Time Slope Only
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                       ///
      c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess    ///
      c.reas22 c.reas22#c.c1sess, || id: c1sess c1sess2,     ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
```

```
        estat recovariance, level(id),
        estimates store Reas2,            // save LL for LRT
        lrtest Reas2 Age,                 // LRT against age baseline
        margins, at(c.c1sess=(0(1)5)) dydx(c.reas22) vsquish     // reas slope per session
        margins, at(c.c1sess=(0(1)5) c.reas22=(-5 0 5)) vsquish  // predictions per session
        marginsplot, name(predicted_reas, replace)              // plot reas predictions
        predict predreas2                 // save fixed-effect predicted outcomes
corr nm3rt predreas2                      // get total r to make r2
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =       606
Group variable: id                              Number of groups   =       101
                                                Obs per group: min =         6
                                                               avg =       6.0
                                                               max =         6
                                                Wald chi2(7)       =    101.09
Log likelihood =  -4150.032                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------
               nm3rt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
              c1sess |  -123.5416   19.82897    -6.23   0.000    -162.4057   -84.67758
     c.c1sess#c.c1sess |   13.97744   3.375697     4.14   0.000       7.3612   20.59369
               age80 |   20.84705   8.561395     2.44   0.015     4.067021    37.62707
      c.age80#c.c1sess |  -4.860993   3.290742    -1.48   0.140    -11.31073    1.588743
c.age80#c.c1sess#c.c1sess |   .6709122   .5580948     1.20   0.229    -.4229335    1.764758
              reas22 |  -32.82806   10.47071    -3.14   0.002    -53.35027   -12.30585
     c.reas22#c.c1sess |    2.93618   1.241355     2.37   0.018     .5031693    5.369191
               _cons |   1969.802    49.6827    39.65   0.000     1872.425    2067.178
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------  Relative to age-only model:
id: Unstructured             |
                 var(c1sess) |   24876.74   5713.188      15860.12    39019.37 → linear var up by -2.40%
                var(c1sess2) |   606.3538   167.7562       352.557    1042.852 → quad var not reduced
                  var(_cons) |   228693.2   34638.71      169952.4    307736.8 → intercept var down by 5.68%
          cov(c1sess,c1sess2) |  -3767.223   957.7847     -5644.446   -1889.999
            cov(c1sess,_cons) |  -31963.12   10883.66     -53294.71   -10631.53
           cov(c1sess2,_cons) |   3878.292   1787.961      373.9525    7382.631
-----------------------------+------------------------------------------------
               var(Residual) |   20298.14   1649.108      17310.16     23801.9 → residual var not reduced
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(6) =    830.58   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
.         estat ic, n(101),
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    101           .   -4150.032     15     8330.064    8369.291
-----------------------------------------------------------------------------
               Note:  N=101 used in calculating BIC

.         estimates store Reas2,          // save LL for LRT
.         lrtest Reas2 Age,               // LRT against age baseline

Likelihood-ratio test                      LR chi2(2)  =      10.14
(Assumption: Age nested in Reas2)          Prob > chi2 =     0.0063
```

Is the revised reasoning model (5b) still better than the age model (4a)?

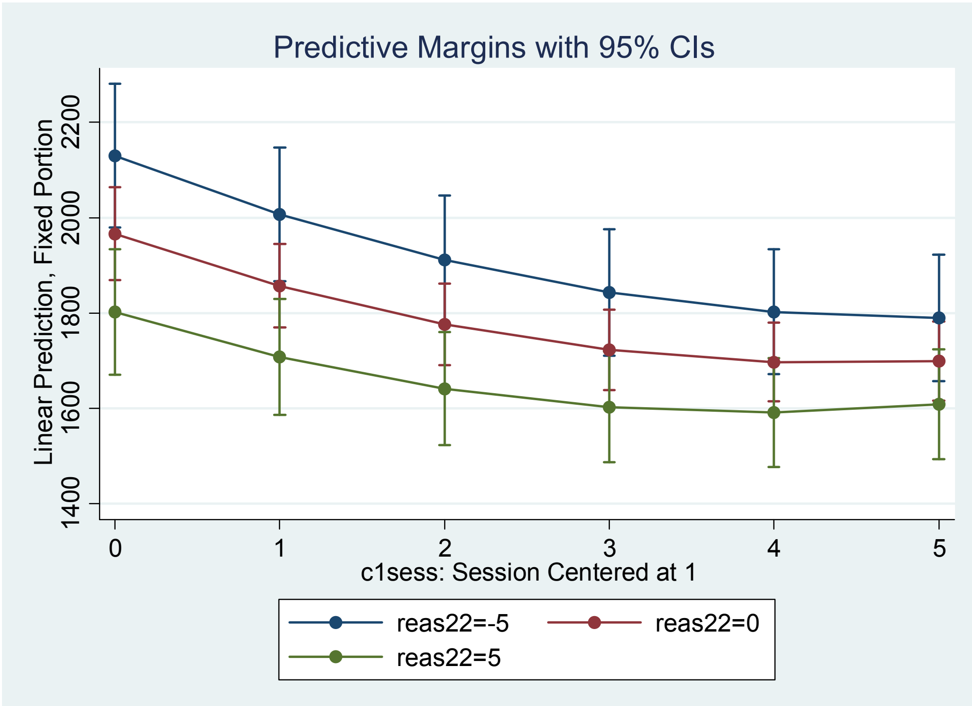
Yes, −2ΔLL = 10.1 on df=2, *p*=.006 (so only 2.4 of the previous −2ΔLL was due to reason*quad)

```
.         margins, at(c1sess=(0(1)5)) dydx(reas22) vsquish     // reas slope per session

Average marginal effects                          Number of obs    =        606

Expression   : Linear prediction, fixed portion, predict()
```

```
dy/dx w.r.t. : reas22
1._at        : c1sess          =           0
2._at        : c1sess          =           1
3._at        : c1sess          =           2
4._at        : c1sess          =           3
5._at        : c1sess          =           4
6._at        : c1sess          =           5
```

These are the simple slopes for reasoning at each session.

```
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
reas22   _at |
           1 |  -32.82806   10.47071    -3.14   0.002    -53.35027   -12.30585
           2 |  -29.89188   9.961528    -3.00   0.003    -49.41612   -10.36764
           3 |   -26.9557   9.586986    -2.81   0.005    -45.74585   -8.165552
           4 |  -24.01952   9.363252    -2.57   0.010    -42.37116   -5.667884
           5 |  -21.08334   9.301214    -2.27   0.023    -39.31338   -2.853295
           6 |  -18.14716   9.404074    -1.93   0.054    -36.57881    .2844865
------------------------------------------------------------------------------

.         margins, at(c1sess=(0(1)5) reas22=(-5 0 5)) vsquish  // predictions per session

Predictive margins                                Number of obs   =        606

Expression   : Linear prediction, fixed portion, predict()
1._at        : c1sess          =           0
               reas22          =          -5
2._at        : c1sess          =           0
               reas22          =           0
3._at        : c1sess          =           0
               reas22          =           5
(output continues for all other sessions)

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
           1 |   2130.467   76.79056    27.74   0.000      1979.96    2280.974
           2 |   1966.327   49.71952    39.55   0.000     1868.878    2063.775
           3 |   1802.186   67.29827    26.78   0.000     1670.284    1934.089
           4 |    2006.92   71.31601    28.14   0.000     1867.143    2146.697
           5 |   1857.461    44.5668    41.68   0.000     1770.112     1944.81
           6 |   1708.002   62.03248    27.53   0.000      1586.42    1829.583
           7 |   1911.105   69.07653    27.67   0.000     1775.717    2046.492
           8 |   1776.326   43.59481    40.75   0.000     1690.882    1861.771
           9 |   1641.548   60.20767    27.26   0.000     1523.543    1759.553
          10 |   1843.021   67.77101    27.19   0.000     1710.192    1975.849
          11 |   1722.923    43.0615    40.01   0.000     1638.524    1807.322
          12 |   1602.825   59.15403    27.10   0.000     1486.886    1718.765
          13 |   1802.668   66.80286    26.98   0.000     1671.736    1933.599
          14 |   1697.251   41.95444    40.45   0.000     1615.022     1779.48
          15 |   1591.834   58.16663    27.37   0.000      1477.83    1705.839
          16 |   1790.046   67.57664    26.49   0.000     1657.598    1922.494
          17 |    1699.31   42.47414    40.01   0.000     1616.062    1782.558
          18 |   1608.574    58.8501    27.33   0.000      1493.23    1723.918
------------------------------------------------------------------------------
```




```
.         marginsplot, name(predicted_reas, replace)                    // plot reas predictions
  Variables that uniquely identify margins: c1sess reas22

.         predict predreas2                       // save fixed-effect predicted outcomes
(option xb assumed)

. corr nm3rt predreas2                        // get total r to make r2
(obs=606)

             |    nm3rt predre~2
-------------+------------------
       nm3rt |   1.0000
   predreas2 |   0.4001   1.0000
```

R =.4001, so R^2 for time+age+reas = .1601

So ~0.1% of the variance accounted for previously was due to reason*quad

Model 6a. +Education Group on Intercept, Linear, and Quadratic Time Slopes

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + \gamma_{03}\left(\text{Highvs.LowEd}_i\right) + \gamma_{04}\left(\text{Highvs.MedEd}_i\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + \gamma_{13}\left(\text{Highvs.LowEd}_i\right) + \gamma_{14}\left(\text{Highvs.MedEd}_i\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{23}\left(\text{Highvs.LowEd}_i\right) + \gamma_{24}\left(\text{Highvs.MedEd}_i\right) + U_{2i}$

Additional model-implied group differences:

Medium vs. Low education intercept $= (\gamma_{00} + \gamma_{04}) - (\gamma_{00} + \gamma_{03}) = \gamma_{04} - \gamma_{03}$

Medium vs. Low education linear session $= (\gamma_{10} + \gamma_{14}) - (\gamma_{10} + \gamma_{13}) = \gamma_{14} - \gamma_{13}$

Medium vs. Low education quadratic session $= (\gamma_{20} + \gamma_{24}) - (\gamma_{20} + \gamma_{23}) = \gamma_{24} - \gamma_{23}$

```
TITLE1 "SAS Model 6a: +Education Group on Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session educgrp;
      MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                    reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
                          / SOLUTION DDFM=Satterthwaite OUTPM=work.EducPred;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
      REPEATED session / TYPE=VC SUBJECT=ID;
      * Estimating group means at first and last sessions
      LSMEANS educgrp  / AT (c1sess) = (0) DIFF=ALL;
      LSMEANS educgrp  / AT (c1sess) = (5) DIFF=ALL;
      * Contrasts between groups on intercept, linear, and quadratic slopes
      ESTIMATE "L vs. H Educ for Intercept Main Effect"  educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Intercept Main Effect"  educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Intercept Main Effect"  educgrp -1  1  0 ;
      ESTIMATE "L vs. H Educ for Linear Session"  c1sess*educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Linear Session"  c1sess*educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Linear Session"  c1sess*educgrp -1  1  0 ;
      ESTIMATE "L vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  1  0 ;
RUN; PROC CORR NOSIMPLE DATA=work.EducPred; VAR nm3rt pred; RUN;
```

Think of the −1 as the "0" and the "1" as the "1" in a dummy code.

```
TITLE "SPSS Model 6a: +Education as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session educgrp WITH c1sess age80 reas22
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (prededuc)
      /EMMEANS = TABLES(educgrp) WITH (c1sess=0) COMPARE(educgrp)
      /EMMEANS = TABLES(educgrp) WITH (c1sess=5) COMPARE(educgrp)
      /TEST = "L vs. H Educ for for Main Effect" educgrp -1  0  1
      /TEST = "M vs. H Educ for for Main Effect" educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Main Effect" educgrp -1  1  0
      /TEST = "L vs. H Educ for for Linear Session" c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Linear Session" c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Linear Session" c1sess*educgrp -1  1  0
      /TEST = "L vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  1  0.
CORRELATIONS nm3rt prededuc.
```

```
* STATA Model 6a: +Education Group on Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                             ///
      c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess             ///
      c.reas22 c.reas22#c.c1sess                                      ///
      b(last).educgrp ib(last).educgrp#c.c1sess                       ///
      ib(last).educgrp#c.c1sess#c.c1sess, || id: c1sess c1sess2,      ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store Educ,
      lrtest Educ Reas2,
 * Estimating group means at first and last sessions
      margins ib(last).educgrp,  at(c.c1sess=(0 5))
* Contrasts between groups on intercept, linear, and quadratic slopes
      test 1.educgrp=3.educgrp                         // Low vs. High: Intercept
      test 2.educgrp=3.educgrp                         // Med vs. High: Intercept
      test 1.educgrp=2.educgrp                         // Low vs. Med:  Intercept
      test 1.educgrp#c.c1sess=3.educgrp#c.c1sess              // Low vs. High: Linear
      test 2.educgrp#c.c1sess=3.educgrp#c.c1sess              // Med vs. High: Linear
      test 1.educgrp#c.c1sess=2.educgrp#c.c1sess              // Low vs. Med:  Linear
      test 1.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Low vs. High: Quad
      test 2.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Med vs. High: Quad
      test 1.educgrp#c.c1sess#c.c1sess=2.educgrp#c.c1sess#c.c1sess  // Low vs. Med:  Quad
```

```
    contrast educgrp,                                    // omnibus group diff on intercept
    contrast educgrp#c.c1sess,                           // omnibus group diff on linear
    contrast educgrp#c.c1sess#c.c1sess,                  // omnibus group diff on quadratic
    margins, at(c.c1sess=(0(1)5) educgrp=(1 2 3)) vsquish  // predictions per session
    marginsplot, name(predicted_educ, replace)           // plot educ predictions
    predict prededuc                    // save fixed-effect predicted outcomes
corr nm3rt prededuc                     // get total r to make r2
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =       606
Group variable: id                              Number of groups   =       101
                                                Obs per group: min =         6
                                                               avg =       6.0
                                                               max =         6
                                                Wald chi2(13)      =    106.94
Log likelihood = -4147.6829                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------------------
                   nm3rt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------------
                  c1sess |  -106.4987   40.28349    -2.64   0.008    -185.4529   -27.54452
       c.c1sess#c.c1sess |   12.47966   6.848972     1.82   0.068    -.9440805     25.9034
                   age80 |   20.28963   8.560341     2.37   0.018     3.511673    37.06759
          c.age80#c.c1sess |  -4.575964   3.267261    -1.40   0.161    -10.97968    1.827749
c.age80#c.c1sess#c.c1sess |   .6176862   .5534216     1.12   0.264    -.4670003    1.702373
                  reas22 |  -36.62127   10.76417    -3.40   0.001    -57.71865   -15.52389
        c.reas22#c.c1sess |   2.978327   1.280262     2.33   0.020     .4690609    5.487594
                 educgrp |
                       1 |  -51.37682   151.0698    -0.34   0.734    -347.4683    244.7146
                       2 |   37.64254   120.8739     0.31   0.755    -199.2659    274.5509
          educgrp#c.c1sess |
                       1 |  -70.24589   59.07811    -1.19   0.234    -186.0368    45.54507
                       2 |  -4.357662   48.13262    -0.09   0.928    -98.69587    89.98055
educgrp#c.c1sess#c.c1sess |
                       1 |   11.06526   10.03239     1.10   0.270    -8.597857    30.72837
                       2 |  -1.464111   8.188464    -0.18   0.858    -17.51321    14.58498
                   _cons |   1961.886   101.7896    19.27   0.000     1762.382     2161.39
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(c1sess) |   24143.68    5618.86      15300.54    38097.82 → linear var down by 2.95%
                var(c1sess2) |   582.0323   164.4561      334.5305    1012.648 → quad var down by 4.01%
                  var(_cons) |   228602.6   34699.74      169776.2    307812.1 → intercept var down by 0.04%
         cov(c1sess,c1sess2) |  -3636.004   939.9484     -5478.269   -1793.739
           cov(c1sess,_cons) |  -33285.68   10917.47     -54683.52   -11887.83
          cov(c1sess2,_cons) |   4127.595   1789.937       619.382    7635.808
-----------------------------+------------------------------------------------
               var(Residual) |   20298.12   1649.105      17310.14    23801.87 → residual var not reduced
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(6) =   826.31   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.

.         estat ic, n(101),

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    101           .   -4147.683     21     8337.366    8392.283
-----------------------------------------------------------------------------
               Note:  N=101 used in calculating BIC
```

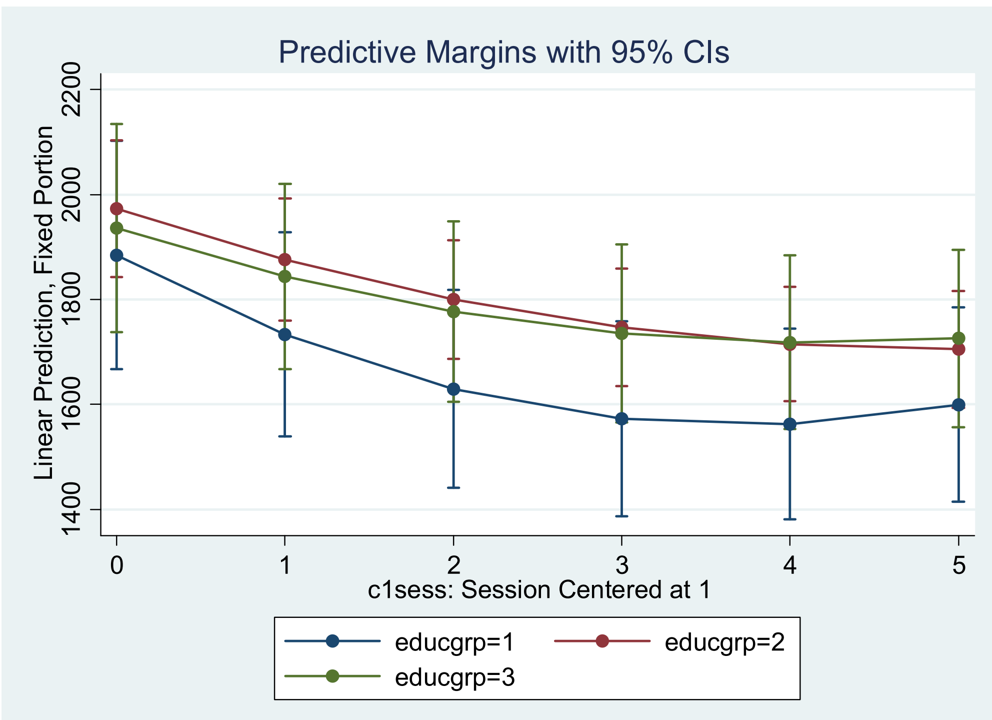
Is the education model (6a) better than the revised reasoning model (5b)?
No, −2ΔLL = 4.7 on df=6, p = .583

```
.        estimates store Educ,
.        lrtest Educ Reas2,

Likelihood-ratio test                                 LR chi2(6)  =      4.70
(Assumption: Reas2 nested in Educ)                    Prob > chi2 =    0.5831

.  * Estimating group means at first and last sessions
.        margins ib(last).educgrp,  at(c1sess=(0))
Predictive margins                                Number of obs   =        606
Expression   : Linear prediction, fixed portion, predict()
at           : c1sess         =           0
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     educgrp |
          1  |   1884.284   110.9362    16.99   0.000     1666.853    2101.715
          2  |   1973.304   66.56898    29.64   0.000     1842.831    2103.776
          3  |   1935.661   101.2482    19.12   0.000     1737.218    2134.104
------------------------------------------------------------------------------
.        margins ib(last).educgrp,  at(c1sess=(5))
Predictive margins                                Number of obs   =        606
Expression   : Linear prediction, fixed portion, predict()
at           : c1sess         =           5
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     educgrp |
          1  |   1599.713   94.54367    16.92   0.000      1414.41    1785.015
          2  |   1704.939    56.6088    30.12   0.000     1593.988     1815.89
          3  |   1725.687   86.06347    20.05   0.000     1557.006    1894.369
------------------------------------------------------------------------------

.  * Contrasts between groups on intercept, linear, and quadratic slopes
.        test 1.educgrp=3.educgrp                              // Low vs. High: Intercept
 ( 1)  [nm3rt]1.educgrp - [nm3rt]3b.educgrp = 0
           chi2(  1) =    0.12
         Prob > chi2 =    0.7338

.        test 2.educgrp=3.educgrp                              // Med vs. High: Intercept
 ( 1)  [nm3rt]2.educgrp - [nm3rt]3b.educgrp = 0
           chi2(  1) =    0.10
         Prob > chi2 =    0.7555

.        test 1.educgrp=2.educgrp                              // Low vs. Med:  Intercept
 ( 1)  [nm3rt]1.educgrp - [nm3rt]2.educgrp = 0
           chi2(  1) =    0.46
         Prob > chi2 =    0.4960

.        test 1.educgrp#c.c1sess=3.educgrp#c.c1sess            // Low vs. High: Linear
 ( 1)  [nm3rt]1.educgrp#c.c1sess - [nm3rt]3b.educgrp#co.c1sess = 0
           chi2(  1) =    1.41
         Prob > chi2 =    0.2344

.        test 2.educgrp#c.c1sess=3.educgrp#c.c1sess            // Med vs. High: Linear
 ( 1)  [nm3rt]2.educgrp#c.c1sess - [nm3rt]3b.educgrp#co.c1sess = 0
           chi2(  1) =    0.01
         Prob > chi2 =    0.9279

.        test 1.educgrp#c.c1sess=2.educgrp#c.c1sess            // Low vs. Med:  Linear
```

```
 ( 1)  [nm3rt]1.educgrp#c.c1sess - [nm3rt]2.educgrp#c.c1sess = 0
           chi2(  1) =    1.69
         Prob > chi2 =    0.1939

.         test 1.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Low vs. High: Quad
 ( 1)  [nm3rt]1.educgrp#c.c1sess#c.c1sess - [nm3rt]3b.educgrp#co.c1sess#co.c1sess = 0

           chi2(  1) =    1.22
         Prob > chi2 =    0.2700

.         test 2.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Med vs. High: Quad
 ( 1)  [nm3rt]2.educgrp#c.c1sess#c.c1sess - [nm3rt]3b.educgrp#co.c1sess#co.c1sess = 0
           chi2(  1) =    0.03
         Prob > chi2 =    0.8581

.         test 1.educgrp#c.c1sess#c.c1sess=2.educgrp#c.c1sess#c.c1sess  // Low vs. Med:  Quad
 ( 1)  [nm3rt]1.educgrp#c.c1sess#c.c1sess - [nm3rt]2.educgrp#c.c1sess#c.c1sess = 0
           chi2(  1) =    2.12
         Prob > chi2 =    0.1454

.         contrast educgrp,                                            // omnibus group diff on intercept

Contrasts of marginal linear predictions
Margins      : asbalanced
------------------------------------------------
             |         df        chi2     P>chi2
-------------+----------------------------------
nm3rt        |
     educgrp |          2        0.48     0.7869
------------------------------------------------

.         contrast educgrp#c.c1sess,                               // omnibus group diff on linear
Contrasts of marginal linear predictions
Margins      : asbalanced
----------------------------------------------------
                 |         df        chi2     P>chi2
-----------------+----------------------------------
nm3rt            |
educgrp#c.c1sess |          2        1.92     0.3827
----------------------------------------------------

.         contrast educgrp#c.c1sess#c.c1sess,                   // omnibus group diff on quadratic
Contrasts of marginal linear predictions
Margins      : asbalanced
-------------------------------------------------------------
                          |         df        chi2     P>chi2
--------------------------+----------------------------------
nm3rt                     |
educgrp#c.c1sess#c.c1sess |          2        2.18     0.3358
-------------------------------------------------------------

.         margins, at(c.c1sess=(0(1)5) educgrp=(1 2 3)) vsquish  // predictions per session
Predictive margins                                Number of obs   =        606
Expression   : Linear prediction, fixed portion, predict()
1._at        : c1sess          =           0
               educgrp         =           1
2._at        : c1sess          =           0
               educgrp         =           2
3._at        : c1sess          =           0
               educgrp         =           3
(output continues for all other sessions)
```

_at	Margin	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
1	1884.284	110.9362	16.99	0.000	1666.853	2101.715
2	1973.304	66.56898	29.64	0.000	1842.831	2103.776
3	1935.661	101.2482	19.12	0.000	1737.218	2134.104
4	1733.602	98.99478	17.51	0.000	1539.576	1927.628
5	1875.98	59.24278	31.67	0.000	1759.867	1992.094
6	1844.159	90.05891	20.48	0.000	1667.647	2020.672
7	1629.804	96.28618	16.93	0.000	1441.086	1818.521
8	1800.482	57.64972	31.23	0.000	1687.491	1913.474
9	1777.411	87.64532	20.28	0.000	1605.63	1949.193
10	1572.889	94.93902	16.57	0.000	1386.812	1758.967
11	1746.809	56.86718	30.72	0.000	1635.352	1858.267
12	1735.417	86.46259	20.07	0.000	1565.953	1904.88
13	1562.859	92.79589	16.84	0.000	1380.983	1744.736
14	1714.961	55.5437	30.88	0.000	1606.098	1823.825
15	1718.175	84.43878	20.35	0.000	1552.678	1883.672
16	1599.713	94.54367	16.92	0.000	1414.41	1785.015
17	1704.939	56.6088	30.12	0.000	1593.988	1815.89
18	1725.687	86.06347	20.05	0.000	1557.006	1894.369

```
.         marginsplot, name(predicted_educ, replace)                       // plot educ predictions
  Variables that uniquely identify margins: c1sess educgrp
```




```
.         predict prededuc                                  // save fixed-effect predicted outcomes
(option xb assumed)

. corr nm3rt prededuc                                       // get total r to make r2
(obs=606)
             |    nm3rt prededuc
-------------+------------------
       nm3rt |   1.0000
    prededuc |   0.4151   1.0000
```

R =.4151, so R^2 for time+age+reas+educ = .172

The fixed effects of time, age, and reasoning before accounted for ~16.0% of the variance in RT, so there is a net increase of 1.2% due to education (which is not significant).

Simple Processing Speed – Example Conditional Models of Change Results

The extent to which individual differences in response time (RT) over six sessions for a simple processing speed test (number match three) could be predicted from baseline age, abstract reasoning, and education level was examined in a series of multilevel models (i.e., general linear mixed models) in which the six practice sessions were nested within each participant. Given the interest in comparing models differing in fixed effects, maximum likelihood (ML) was used in estimating and reporting all model parameters. The significance of new fixed effects were evaluated with individual Wald tests (i.e., of estimate / SE) as well as with likelihood ratio tests (i.e., −2ΔLL), with degrees of freedom equal to the number of new fixed effects. Session (i.e., the index of time) was centered at the first occasion, age was centered at 80 years, abstract reasoning was centered at 22 (near the mean of the scale), and graduate-level education was the reference group for education level (with separate contrasts for high school or less and for bachelor's level education).

The best-fitting unconditional growth model specified quadratic decline across the six sessions (i.e., a decelerating negative function) with significant individual differences in the intercept, linear, and quadratic effects. Accordingly, effect size was evaluated via pseduo-R^2 values for the proportion reduction in each random effect variance, as well as with total R^2, the squared correlation between the actual outcome values and the outcomes predicted by the model fixed effects. In the unconditional growth model, the fixed effects for linear and quadratic change across sessions accounted for approximately 4% of the total variation in RT.

Next, age was added as a predictor of the intercept, linear slope, and quadratic slope. The age model fit significantly better than the unconditional model as indicated by a significant likelihood ratio test, −2ΔLL(3) = 11.6, *p* = .009; the AIC was lower, although the BIC was not. However, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher by 29.05 (*p* = .001). In terms of pseudo-R^2, age accounted for 11.29% of the random intercept variance, 4.50% of the random linear slope variance, and 2.64% of the random quadratic slope variance. As expected given that baseline age is a time-invariant predictor, the residual variance was not reduced. The total cumulative R^2 from session and age was R^2 = .11, approximately a 7% increase due to age (which was significant, as indicated by the likelihood ratio test). Although the interactions of age with the linear and quadratic slopes were not significant, they were retained in the model to fully control for age effects before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept, linear slope, and quadratic slope. The abstract reasoning model fit significantly better than the age model, −2ΔLL(3) = 12.5, *p* = .006; the AIC was lower, although the BIC was not. However, only the fixed effect of reasoning on the intercept was significant. The nonsignificant effect of reasoning on the quadratic slope was then removed, revealing a significant effect of reasoning on both the intercept and linear slope, such that for every unit higher reasoning above 22, RT at the first session was expected to be lower by 32.82 and the linear rate of improvement in RT (as evaluated at the first session given the quadratic slope) was expected to be less negative by 2.94 (i.e., faster initial RT with less improvement in persons with greater reasoning). These two effects still resulted in a significant improvement in model fit over the age model, −2ΔLL(2) = 10.1, *p* = .006, with a lower AIC and BIC. Reasoning accounted for 5.68% of the random intercept variance but had no measurable reduction of the random linear and quadratic slope variances. The total cumulative R^2 from session, age, and reasoning was R^2 = .16, approximately a 5% increase due to reasoning (which was significant, as indicated by the likelihood ratio test).

Finally, education level (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept, linear slope, and quadratic slope. The education model did not fit significantly better than the reasoning model, −2ΔLL(6) = 4.7, *p* = .583, with a higher AIC and BIC. None of the omnibus main effects of group on the intercept, linear, or quadratic slopes were significant, $\chi^2(2) < 1.92$, *p*'s > .05, and none of the pairwise group comparisons were significant as well. Education accounted for 0.04% of the random intercept variance, 2.95% of the random linear slope variance, and 4.01% of the random quadratic slope variance. The total cumulative R^2 from session, age, reasoning, and education was R^2 = .17, approximately a 1% increase due to education (which was not significant, as indicated by the likelihood ratio test).

(From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)