

# Three-Level Models for Clustered Longitudinal Data

- Topics:
  - **Decomposing variation across three levels in clustered longitudinal data**
  - Unconditional (time only) model specification
  - Conditional (other predictors) model specification
  - Other kinds of three-level designs

# What determines the number of levels?

- **Answer: the model for the outcome variance ONLY**
- How many dimensions of sampling in the outcome?
  - Time within person → 2-level model
  - Time within person within family → 3-level model
  - Time within person within family within country → 4-level model
  - Sampling dimensions may also be crossed instead of nested, or may be modeled with fixed effects if the # units is small
- Need at least one pile of variance per dimension (for 3 levels, that's 2 sets of random effects and a residual)
  - Include whatever predictors you want for each level, but keep in mind that the usefulness of your predictors will be constrained by how much Y variance exists in its relevant sampling dimension

# Empty Means, 3-Level Random Intercept Model

Notation:  $t$  = level-1 time,  $i$  = level-2 person,  $j$  = level-3 group

$$\text{Level 1: } y_{tij} = \beta_{0ij} + e_{tij}$$

Residual = time-specific deviation from person's predicted outcome

$$\text{Level 2: } \beta_{0ij} = \delta_{00j} + U_{0ij}$$

Person Random Intercept = person-specific deviation from group's predicted outcome

$$\text{Level 3: } \delta_{00j} = \gamma_{000} + V_{00j}$$

Fixed Intercept = grand mean (because no predictors yet)

Group Random Intercept = group-specific deviation from fixed intercept

**3 Total Parameters:**

**Model for the Means (1):**

- Fixed Intercept  $\gamma_{00}$

**Model for the Variance (2):**

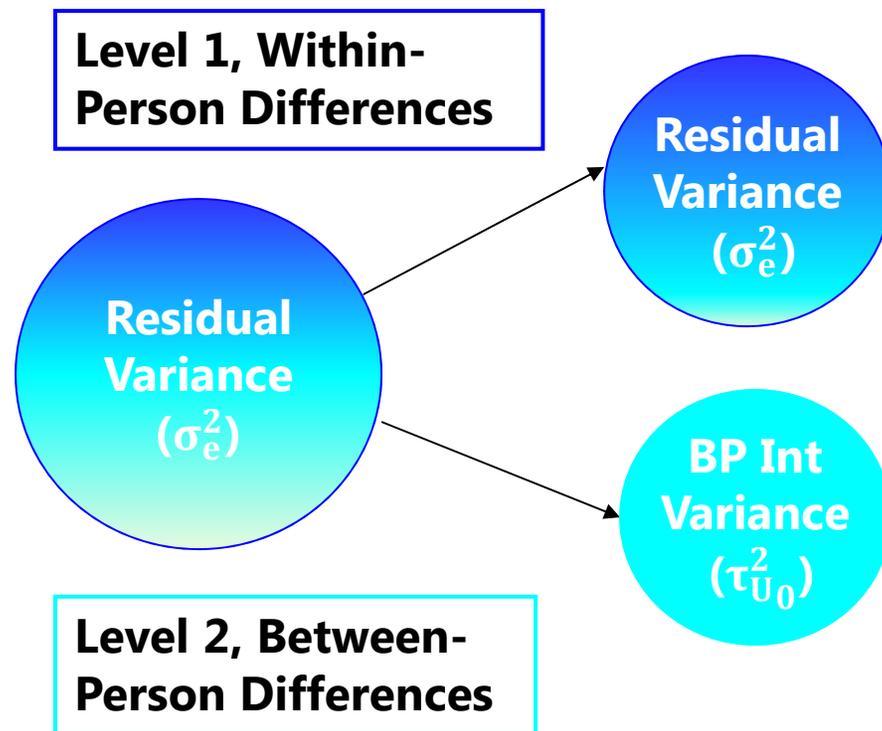
- Level-1 Variance of  $e_{tij} \rightarrow \sigma_e^2$
- Level-2 Variance of  $U_{0ij} \rightarrow \tau_{U_0}^2$
- Level-3 Variance of  $V_{00j} \rightarrow \tau_{V_{00}}^2$

**Composite equation:**

$$y_{tij} = \gamma_{000} + V_{00j} + U_{0ij} + e_{tij}$$

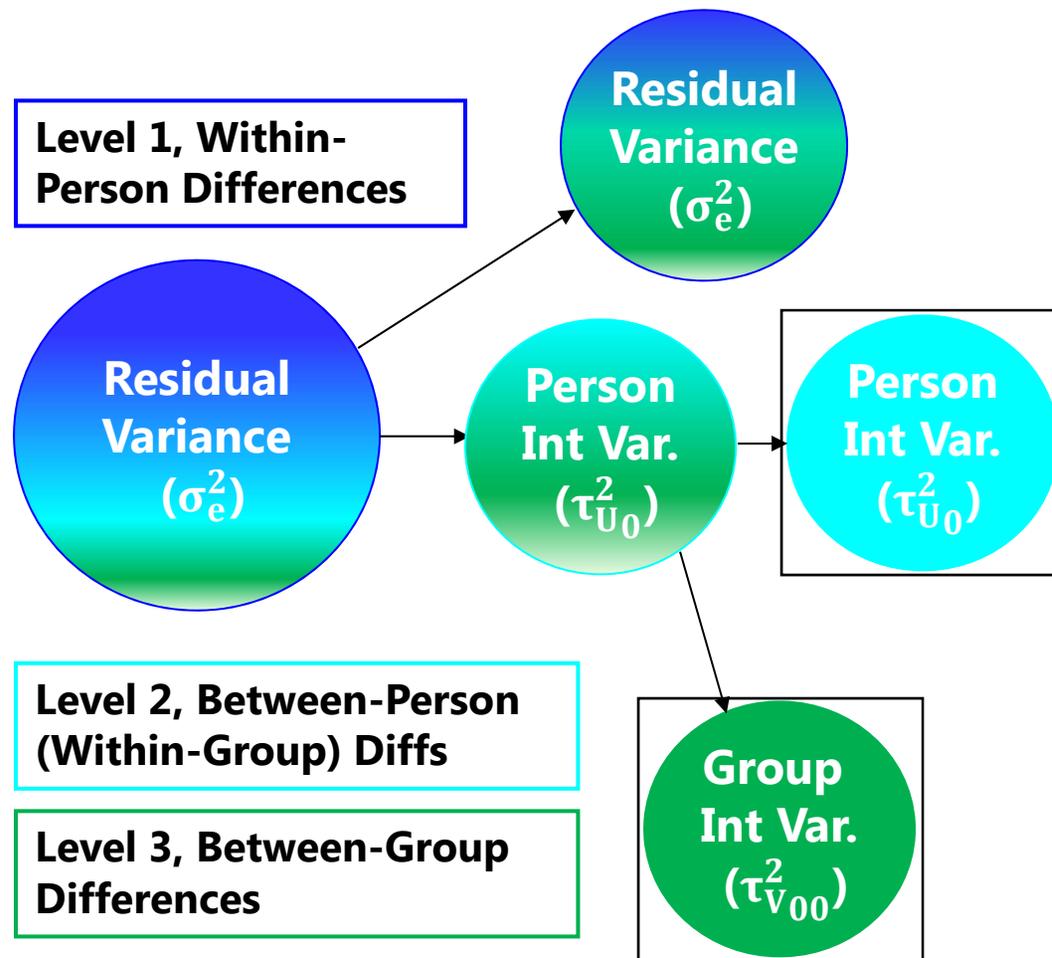
# 2-Level Random Intercept Model

- Where does each kind of person dependency go? Into a new random effects variance component (or “pile” of variance):
- Let’s start with an empty means, random intercept 2-level model for time within person:



# 3-Level Random Intercept Model

- Now let's see what happens in an empty means, random intercept 3-level model of time within person within groups:



# ICCs in a 3-Level Random Intercept Model

## Example: Time within Person within Group

- **ICC for level 2 (and level 3) relative to level 1:**

- $$ICC_{L2} = \frac{\text{Between-Person}}{\text{Total}} = \frac{L3+L2}{L3+L2+L1} = \frac{\tau_{V00}^2 + \tau_{U0}^2}{\tau_{V00}^2 + \tau_{U0}^2 + \sigma_e^2}$$

→ This ICC expresses similarity of occasions from same person (and by definition, from the same group) → of the **total variation in Y**, how much of it is **between persons, or not due to time?**

- **ICC for level 3 relative to level 2 (ignoring level 1):**

- $$ICC_{L3} = \frac{\text{Between-Group}}{\text{Between-Person}} = \frac{L3}{L3+L2} = \frac{\tau_{V00}^2}{\tau_{V00}^2 + \tau_{U0}^2}$$

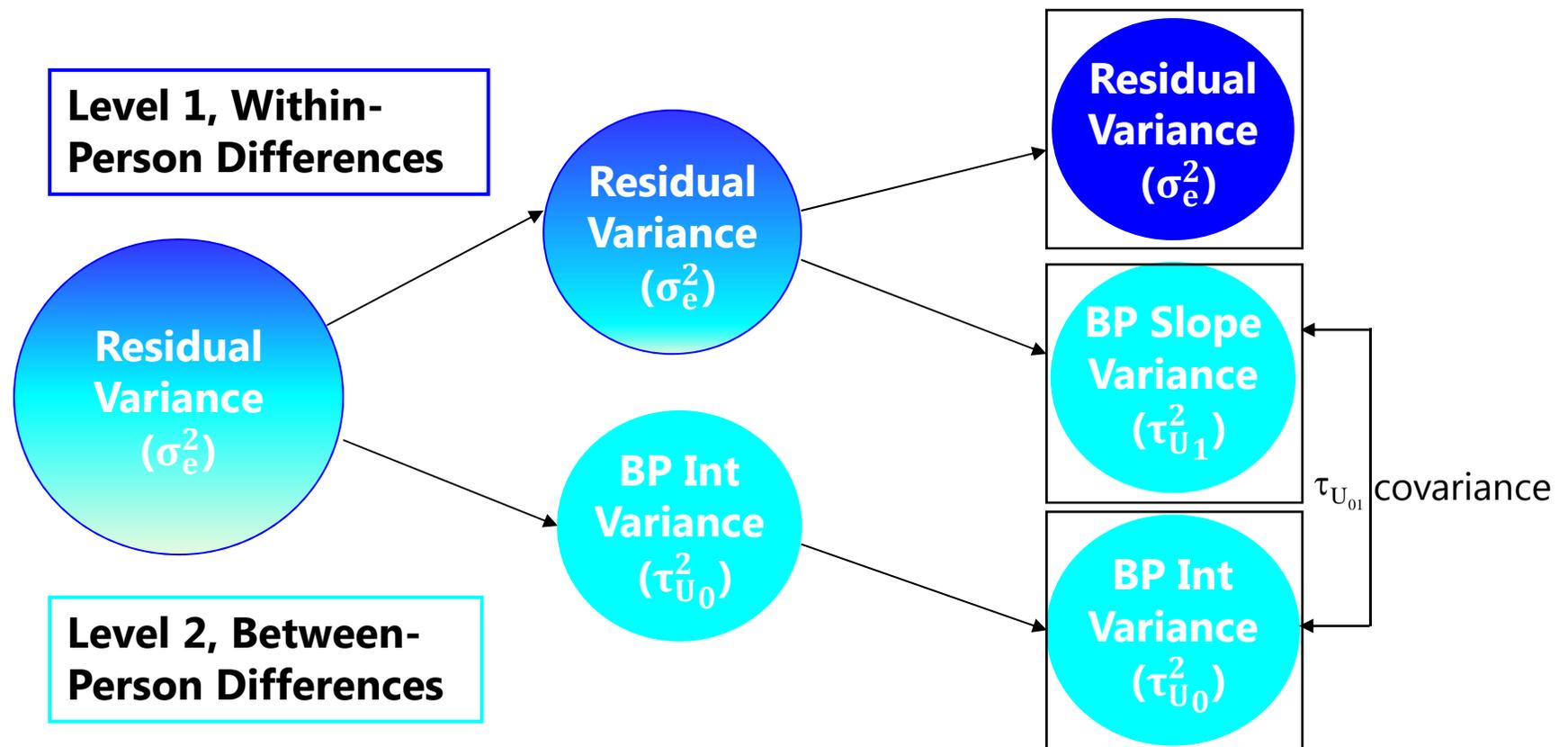
→ This ICC expresses similarity of persons from same group (ignoring within-person variation over time) → of **that total between-person variation in Y**, how much of that is actually **between groups?**

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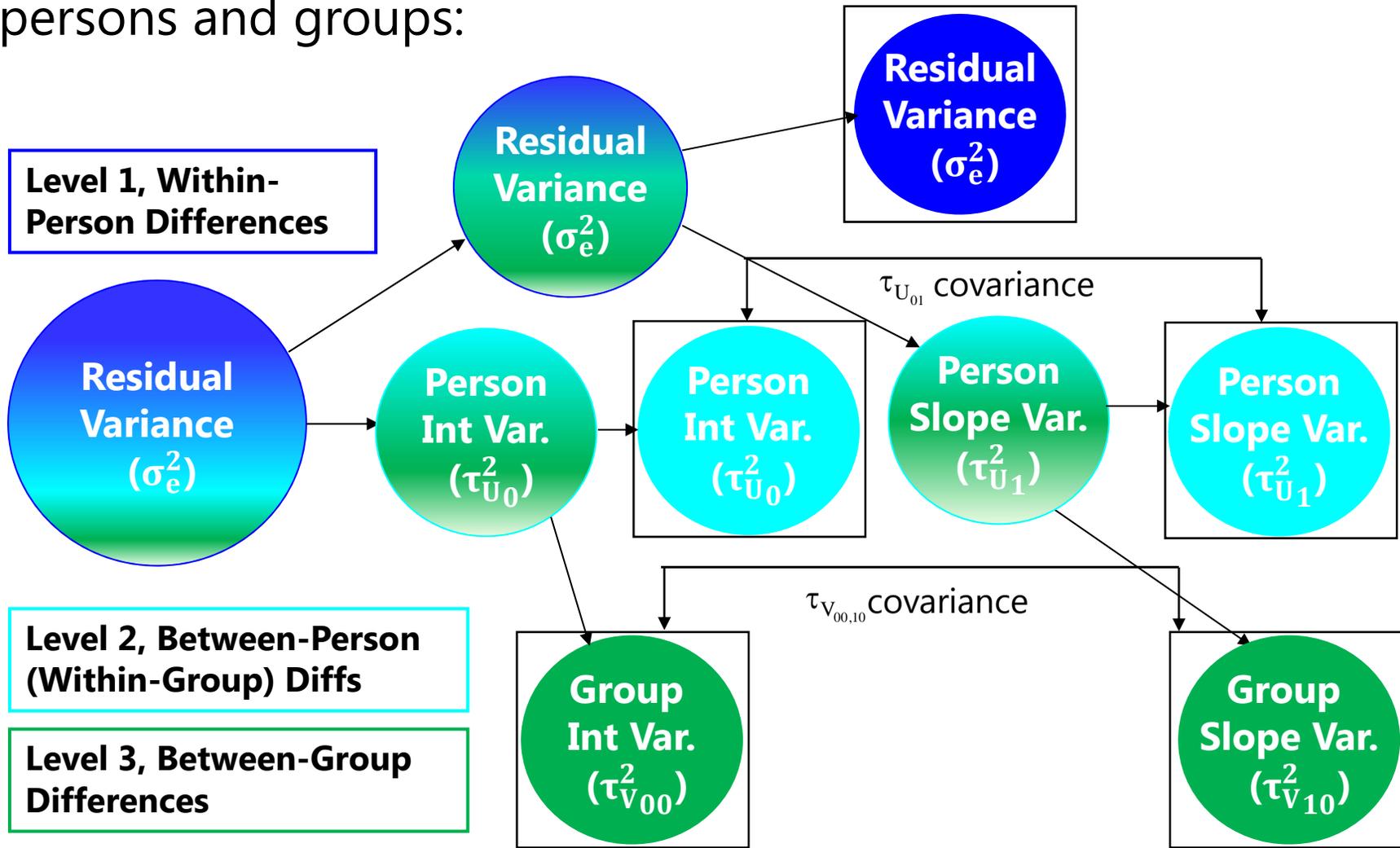
# 2-Level Random Slope Model

- What about time? After adding fixed effects of time, we can add random effects of time over persons in a 2-level model:

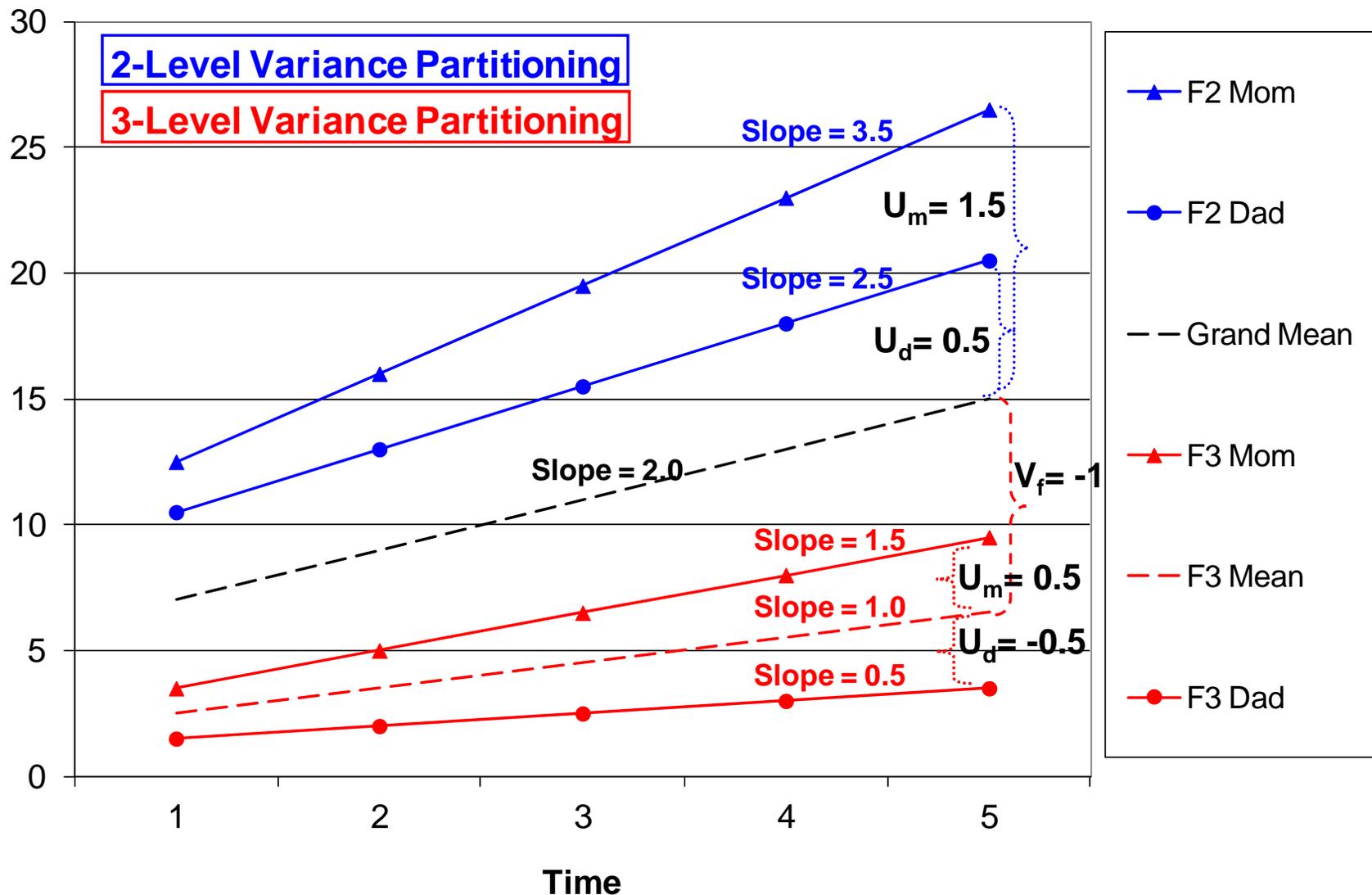


# 3-Level Random Slope Model

- In a 3-level model, we can have random effects of time over persons and groups:



# Random Time Slopes at both Level 2 AND Level 3? An example with family as group:



# 3-Level Random Time Slope Model

Notation:  $t$  = level-1 time,  $i$  = level-2 person,  $j$  = level-3 group

**Level 1:**  $y_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Time}_{tij}) + e_{tij}$  ← Residual = time-specific deviation from person's predicted growth line ( $\sigma_e^2$ )

**Level 2:**  $\beta_{0ij} = \delta_{00j} + U_{0ij}$   
 $\beta_{1ij} = \delta_{10j} + U_{1ij}$  ← Person Random Intercept and Slope = person-specific deviations from group's predicted intercept, slope ( $\tau_{U_0}^2, \tau_{U_1}^2, \tau_{U_{01}}$ )

**Level 3:**  $\delta_{00j} = Y_{000} + V_{00j}$   
 $\delta_{10j} = Y_{100} + V_{10j}$  ← Group Random Intercept and Slope = group-specific deviations from fixed intercept, slope ( $\tau_{V_{00}}^2, \tau_{V_{10}}^2, \tau_{V_{00,10}}$ )

**Fixed Intercept,  
Fixed Linear  
Time Slope**

**Composite equation (9 parameters):**

$$y_{tij} = (Y_{000} + V_{00j} + U_{0ij}) + (Y_{100} + V_{10j} + U_{1ij})(\text{Time}_{tij}) + e_{tij}$$

# ICCs for Random Intercepts and Slopes

- Once random slopes are included at both level-3 and level-2, ICCs can be computed for the random intercepts and slopes specifically (which would be the level-3 type of ICC)

$$ICC_{Int} = \frac{\text{Between} - \text{Group}}{\text{Between} - \text{Person}} = \frac{\text{L3 Int}}{\text{L3 Int} + \text{L2 Int}} = \frac{\tau_{V_{00}}^2}{\tau_{V_{00}}^2 + \tau_{U_0}^2}$$

$$ICC_{Slope} = \frac{\text{Between} - \text{Group}}{\text{Between} - \text{Person}} = \frac{\text{L3 Slope}}{\text{L3 Slope} + \text{L2 Slope}} = \frac{\tau_{V_{10}}^2}{\tau_{V_{10}}^2 + \tau_{U_1}^2}$$

- Can be computed for any level-1 slope that is random at both levels (e.g., linear and quadratic time, time-varying predictors)
- Be careful when the model is uneven across levels, though

$$\frac{\text{Random Level 2: int, linear, quad}}{\text{Random Level 3: int, linear}} \rightarrow \frac{\text{Linear is when time} = 0}{\text{Linear is at any occasion}}$$

# More on Random Slopes in 3-Level Models

- Any level-1 predictor can have a random effect over level 2, level 3, or over both levels, but I recommend working your way **UP the higher levels** for assessing random effects...
  - e.g., Does the effect of time vary over persons?
  - If so, does the effect of time vary over groups, too? → Is there a commonality in how people from the same group change over time?
- ... because random effects at level 3 only are possible but unlikely (e.g., means everyone in the group changes the same)
- Level-2 predictors can also have random effects over level 3
  - e.g., Does the effect of a person characteristic vary over groups?
- Level-1, level-2, and level-1 by level-2 cross-level interactions can all have random effects over level 3, too
  - But tread carefully! The more random effects you have, the more likely you are to have convergence problems (“not positive definite”)

# Three-Level Models for Clustered Longitudinal Data

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# Conditional Model Specification

- Remember separating between- and within-person effects?  
Now there are 3 potential effects for any level-1 predictor!
  - Example: Effect of stress on wellbeing, both measured over time within person within families:
  - **Level 1** (Time): During **Times** of more stress, people have lower (time-specific) wellbeing than in times of less stress
  - **Level 2** (Person): **People** in the family who have more stress have lower (person average) wellbeing than people in the family who have less stress
  - **Level 3** (Family): **Families** who have more stress have lower (family average) wellbeing than families who have less stress
- 2 potential effects for any level-2 predictor, also
  - Example: Effect of baseline level of person coping skills in same design:
  - **Level 2** (Person): **People** in the family who cope better have better (person average) wellbeing than people in the family who cope worse
  - **Level 3** (Family): **Families** who cope better have better (family average) wellbeing than families who cope worse

# Separate Total Effects Per Level Using Person/Group-Mean-Centering

- **Level 1 (Time):** *Time-varying stress relative to person mean*
  - $WP_{stress_{tjj}} = Stress_{tjj} - PersonMeanStress_{ij}$
  - Directly tests if within-person effect  $\neq 0$ ?
  - **Total** within-person effect of having more stress ***than usual***  $\neq 0$ ?
- **Level 2 (Person):** *Person mean stress relative to family*
  - $WF_{stress_{ij}} = PersonMeanStress_{ij} - FamilyMeanStress_j$
  - Directly tests if within-family effect  $\neq 0$ ?
  - **Total** effect of having more stress ***than other family members***  $\neq 0$ ?
- **Level 3 (Family):** *Family mean stress relative to all families (from constant)*
  - $BF_{stress_j} = FamilyMeanStress_j - C$
  - Directly tests if between-family effect  $\neq 0$ ?
  - **Total** effect of having more stress ***than other families***  $\neq 0$ ?

# Separate Total Effects Per Level Using Person/Group-Mean-Centering

Notation:  $t$  = level-1 time,  $i$  = level-2 person,  $j$  = level-3 group  
 PM = person mean, FM = family mean, C = centering constant

$$\text{Level 1: } y_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Time}_{tij}) + \beta_{2ij}(\text{Stress}_{tij} - \text{PMstress}_{ij}) + e_{tij}$$

$$\text{Level 2: } \beta_{0ij} = \delta_{00j} + \delta_{01j}(\text{PMstress}_{ij} - \text{FMstress}_j) + U_{0ij}$$

$$\beta_{1ij} = \delta_{10j} + U_{1ij}$$

$$\beta_{2ij} = \delta_{20j} + (U_{2ij})$$

$$\text{Level 3: } \delta_{00j} = Y_{000} + Y_{001}(\text{FMstress}_j - C) + V_{00j}$$

$$\delta_{01j} = Y_{010} + (V_{01j})$$

$$\delta_{10j} = Y_{100} + V_{10j}$$

$$\delta_{20j} = Y_{200} + (V_{20j})$$

Fixed intercept,  
 Between-family  
 stress main effect

Within-family stress main effect

Time main effect

Within-person stress main effect

# Contextual Effects Per Level Using Grand-Mean-Centering

- **Level 1 (Time):** *Time-varying stress (relative to sample constant)*
  - $TVstress_{tij} = Stress_{tij} - C$
  - Directly tests if within-person effect  $\neq 0$ ?
  - **Total** within-person effect of having more stress **than usual**  $\neq 0$ ?
- **Level 2 (Person):** *Person mean stress (relative to sample constant)*
  - $BPstress_{ij} = PersonMeanStress_{ij} - C$
  - Directly tests if within-person and within-family effects  $\neq ?$
  - **Contextual** effect of having more stress **than other family members**  $\neq 0$ ?
- **Level 3 (Family):** *Family mean stress relative to all families (from constant)*
  - $BFstress_j = FamilyMeanStress_j - C$
  - Directly tests if within-family and between-family effects  $\neq ?$
  - **Contextual** effect of having more stress **than other families**  $\neq 0$ ?

# Contextual Effects Per Level Using Grand-Mean-Centering

Notation:  $t$  = level-1 time,  $i$  = level-2 person,  $j$  = level-3 group  
 PM = person mean, FM = family mean, C = centering constant

**Level 1:**  $y_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Time}_{tij}) + \beta_{2ij}(\text{Stress}_{tij} - C) + e_{tij}$

**Level 2:**  $\beta_{0ij} = \delta_{00j} + \delta_{01j}(\text{PMstress}_{ij} - C) + U_{0ij}$

$\beta_{1ij} = \delta_{10j} + U_{1ij}$

$\beta_{2ij} = \delta_{20j} + (U_{2ij})$

**Level 3:**  $\delta_{00j} = Y_{000} + Y_{001}(\text{FMstress}_j - C) + V_{00j}$

$\delta_{01j} = Y_{010} + (V_{01j})$

$\delta_{10j} = Y_{100} + V_{10j}$

$\delta_{20j} = Y_{200} + (V_{20j})$

**Fixed intercept,  
Contextual family  
stress main effect**

**Contextual within-family stress main effect**

**Time main effect**

**Within-person stress main effect**

# What does it mean to omit higher-level effects under each centering method?

- **Person-MC:** Removing terms means the effect at that level does not exist (= 0)
  - Remove L3 effect? Assume L3 Between-Family effect = 0
    - *L1 effect = Within-Person effect, L2 effect = Within-Family effect*
  - Then remove L2 effect? Assume L2 Within-Family effect = 0
    - *L1 effect = Within-Person effect*
- **Grand-MC:** Removing terms means the effect at that level is equivalent to the effect at the level beneath it
  - Remove L3 effect? Assume L3 Between-Family = L2 Within-Family effect
    - *L1 effect = Within-Person effect, L2 effect = 'smushed' WF and BF effects*
  - Then remove L2 effect? Assume L2 Between-Person effect = L1 effect
    - *L1 'smushed' = Within-Person, Within-Family, and Between-Family effects*

# Interactions belong at each level, too...

- Example: Is the effect of stress on wellbeing moderated by time-invariant person coping? Using person/group-MC...
- **Stress Effects**
  - **Level 1 (Time):**  $WPstress_{tij} = Stress_{tij} - PersonMeanStress_{ij}$
  - **Level 2 (Person):**  $WFstress_{ij} = PersonMeanStress_{ij} - FamilyMeanStress_j$
  - **Level 3 (Family):**  $BFstress_j = FamilyMeanStress_j - C$
- **Coping Effects**
  - **Level 2 (Person):**  $WFcope_{ij} = Cope_{ij} - FamilyMeanCope_j$
  - **Level 3 (Family):**  $BFcope_j = FamilyMeanCope_j - C$
- **Interaction Effects**
  - With level 1 stress:  $WPstress_{tij} * WFcope_{ij}$ ,  $WPstress_{tij} * BFcope_j$
  - With level 2 stress:  $WFstress_{ij} * WFcope_{ij}$ ,  $(WFstress_{ij} * BFcope_j)$
  - With level 3 stress:  $BFstress_j * BFcope_j$ ,  $(BFstress_j * WFcope_{ij})$

# Interactions belong at each level, too...

Notation:  $t$  = level-1 time,  $i$  = level-2 person,  $j$  = level-3 group  
 PM = person mean, FM = family mean, C = centering constant

$$\text{Level 1: } y_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Time}_{tij}) + \beta_{2ij}(\text{Stress}_{tij} - \text{PMstress}_{ij}) + e_{tij}$$

$$\begin{aligned} \text{Level 2: } \beta_{0ij} = & \delta_{00j} + \delta_{01j}(\text{PMstress}_{ij} - \text{FMstress}_j) \\ & + \delta_{02j}(\text{Cope}_{ij} - \text{FMcope}_j) \\ & + \delta_{03j}(\text{PMstress}_{ij} - \text{FMstress}_j)(\text{Cope}_{ij} - \text{FMcope}_j) + U_{0ij} \end{aligned}$$

$$\beta_{1ij} = \delta_{10j} + U_{1ij}$$

$$\beta_{2ij} = \delta_{20j} + \delta_{21j}(\text{Cope}_{ij} - \text{FMcope}_j) + (U_{2ij})$$

$$\begin{aligned} \text{Level 3: } \delta_{00j} = & Y_{000} + Y_{001}(\text{FMstress}_j - C) + Y_{002}(\text{FMcope}_j - C) \\ & + Y_{003}(\text{FMstress}_j - C)(\text{FMcope}_j - C) + V_{00j} \end{aligned}$$

$$\delta_{01j} = Y_{010} + (V_{01j}) \quad \delta_{02j} = Y_{020} + (V_{02j}) \quad \delta_{03j} = Y_{030} + (V_{03j})$$

$$\delta_{10j} = Y_{100} + V_{10j}$$

$$\delta_{20j} = Y_{200} + Y_{202}(\text{FMcope}_j - C) + (V_{20j}) \quad \delta_{21j} = Y_{210} + (V_{21j})$$

# Summary: Clustered Longitudinal Models

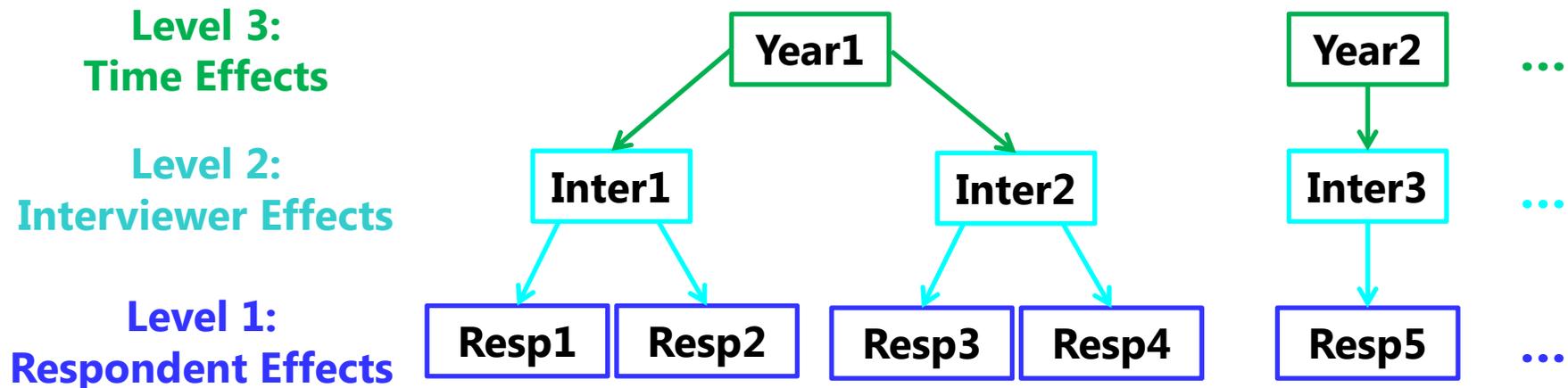
- Estimating 3-level models requires no new concepts, but everything is just at an order of complexity higher:
  - Proportioning variance over 3 levels instead of 2 → 2+ ICCs
  - Random slope variance will come from term directly beneath:
    - Level-2 random slope comes from level-1 residual
    - Level-3 random slope comes from level-2 random slope (or residual)
  - Level-1 effects can be random over level 2, level 3, or both
    - ICCs can be computed for level-1 slopes that are random over both level-2 and level-3 (assuming the L2 and L3 models match)
    - Convergence of level-1 effects should be tested over levels 2 AND 3
  - Level-2 effects can be random over level 3
    - Convergence of level-2 effects should be tested over level 3
  - Level-3 effects cannot be random; no convergence testing needed
  - Phew....

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# Other 3-Level Designs

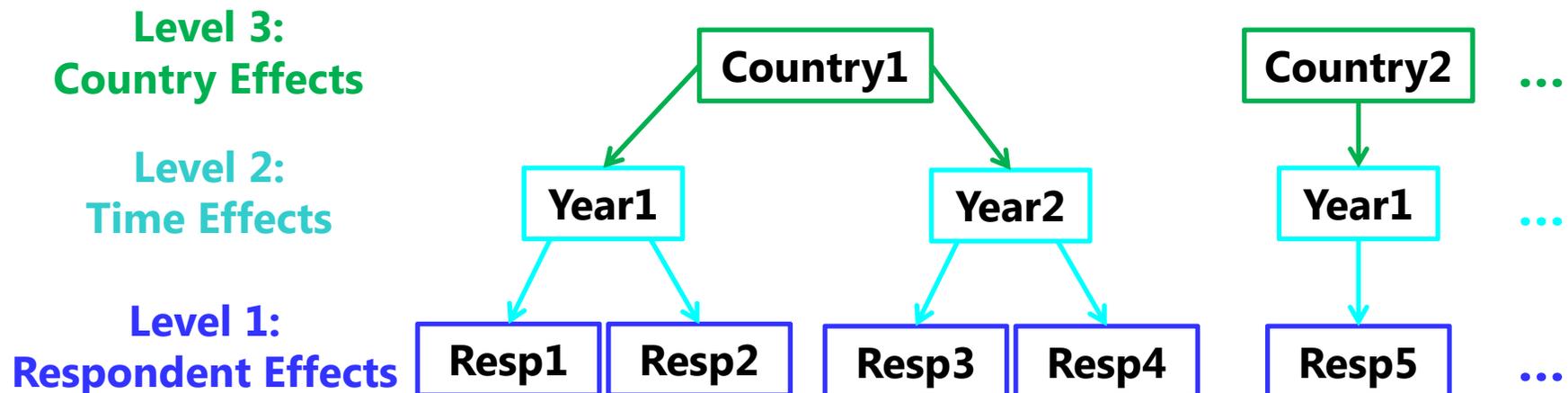
- The sampling design for the outcome (not the predictors) dictates what your levels will be, **so time may not always be level 1**
- Example: Predicting answer compliance in respondents nested in interviewers, collected over several years (all different people)



- Based on the sampling of time, time may be modeled...
  - As fixed effects in the model for the means → 2-level model instead
    - Best to use dummy codes for time if few occasions OR no time-level predictors of interest
  - As a random effect in the model for the variance → 3-level model
    - Then differences in compliance rates over time can be predicted by time-level predictors

# Other 3-Level Designs

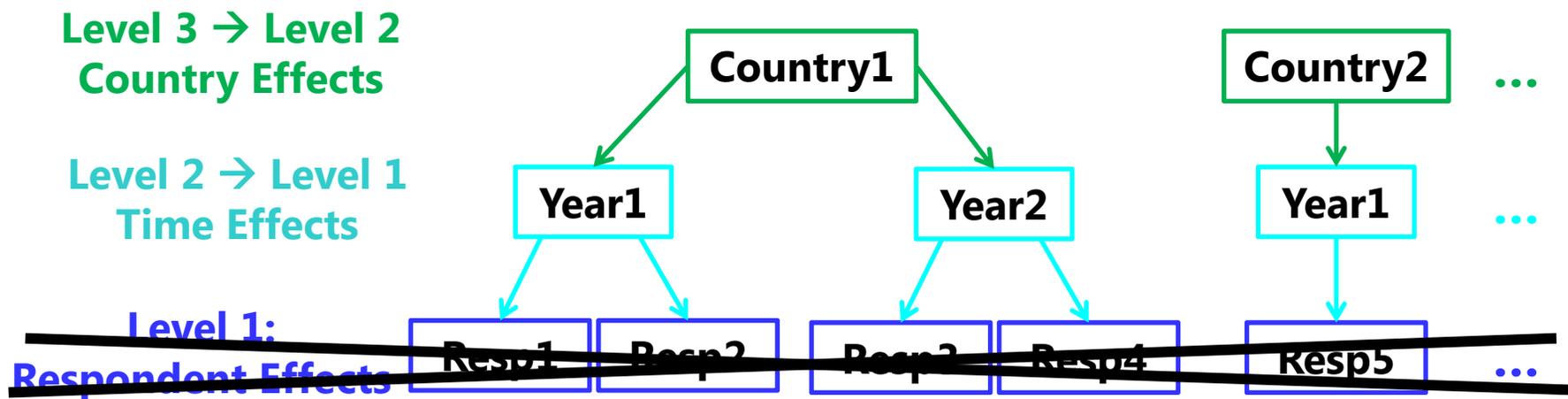
- Another example: Predicting **time-specific respondent outcomes** for people nested in countries, collected over several years (all different people, but the same countries measured over time)



- Before including any fixed effects of time, country and time are actually crossed, not nested as shown here
  - Are nested after controlling for which occasion is which via fixed effects (using dummy codes per mean or a time trend that describes the means)
  - Time is still a level because not all countries change the same way

# 3-Level Designs: Predictors vs. Outcomes

- Same example: What if, instead of respondent outcomes, we wanted to predict **time-varying country outcomes**?

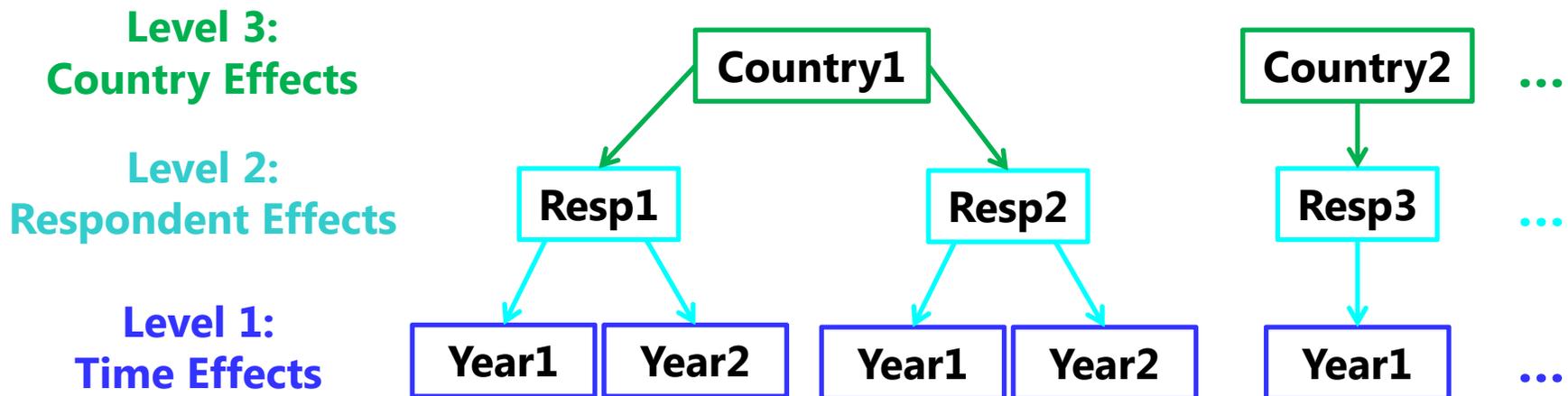


Because the outcome was measured at level 2 (country per time):

- Respondents are no longer a level at all (no outcomes for them)
- So there is nothing for respondent predictors to do, except at higher levels
  - **Time-specific averages** of respondent predictors → time-level outcome variation
  - **Across time, country averages** of respondent predictors → country-level outcome variation

# Other 3-Level Designs: Predictors by Level

- Last example: Predicting **time-specific respondent outcomes** for people nested in countries, collected over several years (all same people and same countries are measured over time)



- Country predictors can be included at level 3 only (no random effects)
- Person predictors should be included at levels 2 and 3 (+random over 3)
- What about effects of time-varying predictors?
  - For People: effects should be included at all 3 levels (+random over 2 and 3)
  - For Countries: effects are only possible at levels 1 and 3 (+random over 3)