Example 5: Two-Level Clustered Data Example: Students within Schools
(only syntax and output available for SAS, SPSS, and STATA electronically)

These are real data taken from the results of a math test given at the end of 10th grade in a Midwestern Rectangular State. These data include 13,802 students from 94 schools, with 31–515 students in each school (M = 275). We will examine how student free and reduced lunch status (0=pay for lunch, 1= receive free or reduced lunch) predicts math test scores.

SAS Code for Data Manipulation:

```
* Importing data into work library;
DATA work.grade10; SET example.grade10;
      * Selecting cases that are complete for analysis variables;
      WHERE NMISS(studentID, schoolID, frlunch, math)=0;
      LABEL  studentID=    "studentID: Student ID number"
             schoolID=     "schoolID: School ID number"
             frlunch=      "frlunch: 0=No, 1=Free/Reduced Lunch"
             math=         "math: Math Test Score Outcome"; RUN;

* Getting school means to use as predictors;
PROC SORT DATA=work.grade10; BY schoolID studentID; RUN;
PROC MEANS NOPRINT N DATA= work.grade10;
      BY schoolID;
      VAR frlunch math;
      OUTPUT OUT=SchoolMeans
             MEAN(frlunch math)= SMfrlunch SMmath; RUN;

* Labeling new school mean variables;
DATA work.SchoolMeans; SET work.SchoolMeans;
      SchoolN = _FREQ_;    * Saving N per school;
      DROP _TYPE_ _FREQ_; * Dropping unneeded SAS-created variables;
      LABEL  SMfrlunch=    "SMfrlunch: School Mean 0=No, 1=Free/Reduced Lunch"
             SMmath=       "SMmath: School Mean Math Outcome"
             SchoolN=      "SchoolN: # Students Contributing Data"; RUN;

* Merging school means back with individual data;
DATA work.grade10; MERGE work.grade10 work.SchoolMeans; BY schoolID;
      * Selecting only schools with data from at least 30 students;
      IF SchoolN < 31 THEN DELETE; RUN;

TITLE "Getting means to center predictors with";
PROC MEANS MEAN STDDEV MIN MAX DATA=work.grade10;
      VAR math frlunch SMmath SMfrlunch SchoolN; RUN; TITLE;

* Centering school mean predictors;
DATA work.grade10; SET work.grade10;
      SMfrlunch30 = SMfrlunch - .30; LABEL SMfrlunch30= "SMfrlunch30: 0=.30"; RUN;
```

SPSS Code for Data Manipulation:

```
* SPSS code to import data and create/center predictors.
DATASET NAME grade10 WINDOW=FRONT.
VARIABLE LABELS
      studentID     "studentID: Student ID number"
      schoolID      "schoolID: School ID number"
      frlunch       "frlunch: 0=No, 1=Free/Reduced Lunch"
      math          "math: Math Test Score".

* Selecting complete cases for analysis.
SELECT IF (NMISS(studentID, schoolID, frlunch, math)=0).
EXECUTE.

* Getting school means to use as level-2 predictors - SPSS 14+ can merge them back automatically.
SORT CASES BY schoolID studentID.
```

```
AGGREGATE
       /OUTFILE=* MODE=ADDVARIABLES
       /PRESORTED
       /BREAK = schoolID
       /SMfrlunch = MEAN(frlunch)
       /SMmath = MEAN(math)
       /SchoolN = N.

* Labeling new school mean variables.
VARIABLE LABELS
       SMfrlunch     "SMfrlunch: School Mean 0=No, 1=Free/Reduced Lunch"
       SMmath        "SMmath: School Mean Math Outcome"
       SchoolN       "SchoolN: # Students Contributing Data".

* Selecting schools with data from at least 30 students.
SELECT IF (SchoolN GT 30).

* Descriptive statistics.
DESCRIPTIVES VARIABLES=math frlunch SMmath SMfrlunch SchoolN
  /STATISTICS=MEAN STDDEV MIN MAX.

* Centering school mean predictor.
COMPUTE SMfrlunch30 = SMfrlunch - .30.
VARIABLE LABELS SMfrlunch30 "SMfrlunch30: 0=.30".
EXECUTE.
```

STATA Code for Data Manipulation:

```
 * label existing variables
label variable studentID "studentID: Student ID number"
label variable schoolID  "schoolID: School ID number"
label variable frlunch   "frlunch: Student Free/Reduced Lunch 0=No 1=Yes"
label variable math      "math: Student Free/Reduced Lunch 0=No 1=Yes"

 * get school means of variables and label them
egen SMfrlunch   = mean(frlunch),   by (schoolID)
egen SMmath      = mean(math),      by (schoolID)
label variable SMfrlunch "SMfrlunch: School Mean 0=No, 1=Free/Reduced Lunch"
label variable SMmath    "SMmath: School Mean Math Outcome"

 * get number of students per school
egen SchoolN = count(studentID), by (schoolID)
label variable SchoolN= "SchoolN: # Students Contributing Data"

 * then drop schools with <= 30 students
drop if SchoolN < 31

* get means to center with
summarize math frlunch SMmath SMfrlunch SchoolN

 * centering school mean predictor
gen SMfrlunch30 = SMfrlunch - .30
label variable SMfrlunch30 "SMfrlunch30: Percentage Students with Free Lunch (0=30%)"
```

Variable	Obs	Mean	Std. Dev.	Min	Max
math	13082	48.11856	17.25905	0	83
frlunch	13082	.3075218	.461485	0	1
SMmath	13082	48.11856	6.81813	29.45098	61.61364
SMfrlunch	13082	.3075218	.2220852	0	.8032787
SchoolN	13082	274.9502	155.3319	31	515

Model 1: Two-Level Empty Means, Random Intercept for Math Test Outcome

$$\text{Level 1: } \text{Math}_{ij} = \beta_{0j} + e_{ij}$$
$$\text{Level 2: } \beta_{0j} = \gamma_{00} + U_{0j}$$

```
TITLE1 "SAS Model 1: 2-Level Empty Means, Random Intercept for Math Outcome";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS schoolID studentID;
       MODEL math = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / TYPE=UN SUBJECT=schoolID; RUN;

TITLE "SPSS Model 1: 2-Level Empty Means, Random Intercept for Math Outcome".
MIXED math BY schoolID studentID
       /METHOD = ML
       /PRINT  = SOLUTION TESTCOV
       /FIXED  =
       /RANDOM = INTERCEPT | SUBJECT(schoolID) COVTYPE(UN).

 * STATA Model 1a: 2-Level Empty Means, Random Intercept for Math Outcome
xtmixed math  , || schoolID: , ///
       variance ml covariance(un) residuals(independent),
       estat ic, n(94)
```

STATA output:

```
Mixed-effects ML regression                     Number of obs      =     13082
Group variable: schoolID                        Number of groups   =        94
                                                Obs per group: min =        31
                                                               avg =     139.2
                                                               max =       515
                                                Wald chi2(0)       =         .
Log likelihood =  -54895.45                     Prob > chi2        =         .
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   47.75613   .7191927    66.40   0.000     46.34654    49.16572
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   44.93635   7.039956      33.05554    61.08735
-----------------------------+------------------------------------------------
               var(Residual) |   253.1756   3.141541      247.0926    259.4084
------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =  1857.08 Prob >= chibar2 = 0.0000
.         estat ic, n(94)
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |     94           .   -54895.45      3     109796.9    109804.5
-----------------------------------------------------------------------------
               Note:  N=94 used in calculating BIC
                  Covariance Parameter Estimates
```

Calculate the ICC for the correlation of students in the same school for math:

$$\text{ICC} = \frac{44.94}{44.94\ +\ 253.18} = .15$$

This LR test tells us that the random intercept variance is significantly greater than 0,

Design effect using mean #students per school: = 1 + ((*n* – 1) * ICC) → 1 + [(275−1)*.15] = **42.1**

Effective sample size: $N_{effective}$ = (#Total Obs) / Design Effect → 13,082 / 42.1 = **311!!!**

95% random effect confidence interval for the intercept across schools: Fixed effect ± 1.96*SQRT(variance)

48 ± 1.96*SQRT(45) = **35 to 61** → 95% of schools are predicted to have school mean math from 35 to 61

Model 2: Adding a Fixed Effect of Student Free/Reduced Lunch (Level 1)

$$\text{Level 1: } \text{Math}_{ij} = \beta_{0j} + \beta_{1j}\left(\text{FRlunch}_{ij}\right) + e_{ij}$$
$$\text{Level 2: } \quad \text{Intercept: } \beta_{0j} = \gamma_{00} + U_{0j}$$
$$\text{Free/Reduced Lunch: } \beta_{1j} = \gamma_{10}$$

```
TITLE1 "SAS Model 2: Adding Fixed Effect of Student Free/Reduced Lunch";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS schoolID studentID;
       MODEL math = frlunch / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / TYPE=UN SUBJECT=schoolID; RUN;

TITLE "SPSS 2: Adding Fixed Effect of Student Free/Reduced Lunch".
MIXED math BY schoolID studentID WITH frlunch
       /METHOD = ML
       /PRINT  = SOLUTION TESTCOV
       /FIXED  = frlunch
       /RANDOM = INTERCEPT | SUBJECT(schoolID) COVTYPE(UN).

 * STATA Model 2: Adding Fixed Effect of Student Free/Reduced Lunch
xtmixed math c.frlunch, || schoolID: , ///
       variance ml covariance(un) residuals(independent),
       estat ic, n(94)
```

STATA output:

```
                                                Wald chi2(1)       =    808.17
Log likelihood = -54508.069                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     frlunch |   -9.43162   .3317684   -28.43   0.000    -10.08187   -8.781366
       _cons |   50.61611   .5766321    87.78   0.000     49.48594    51.74629
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   26.89008   4.439001      19.45701    37.16277 → int var down by 40.16%
-----------------------------+------------------------------------------------
               var(Residual) |   239.3289   2.969964      233.5781    245.2213 → res var down by 5.47%

------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =   891.06 Prob >= chibar2 = 0.0000

.         estat ic, n(94)
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |     94           .   -54508.07      4     109024.1    109034.3
-----------------------------------------------------------------------------
               Note:  N=94 used in calculating BIC
```

What does the effect of student free/reduced lunch represent in this model?
Children who get free/reduced lunch score 9.43 points lower than children who don't.

What are we assuming about the effect of student free/reduced lunch?
We are assuming no contextual effect (that the between-school and within-school effects of FRlunch are equal).

Model 3: Adding a Fixed Effect of School Proportion Free/Reduced Lunch (Level 2)

Level 1: $\text{Math}_{ij} = \beta_{0j} + \beta_{1j}\left(\text{FRlunch}_{ij}\right) + e_{ij}$

Level 2: Intercept: $\beta_{0j} = \gamma_{00} + \gamma_{01}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + U_{0j}$

Free/Reduced Lunch: $\beta_{1j} = \gamma_{10}$

```
TITLE1 "SAS Model 3: Adding Fixed Effect of School Proportion Free/Reduced Lunch";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS schoolID studentID;
      MODEL math = frlunch SMfrlunch30 / SOLUTION DDFM=Satterthwaite OUTPM=work.LunchSave;
      RANDOM INTERCEPT / TYPE=UN SUBJECT=schoolID;
      ESTIMATE "FR Lunch Between-School Effect"  frlunch 1 SMfrlunch30 1;
RUN;
PROC CORR NOSIMPLE DATA=work.LunchSave; VAR math pred; RUN;

TITLE "SPSS Model 3: Adding Fixed Effect of School Proportion Free/Reduced Lunch".
MIXED math BY schoolID studentID WITH frlunch SMfrlunch30
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV
      /FIXED  = frlunch SMfrlunch30
      /RANDOM = INTERCEPT | SUBJECT(schoolID) COVTYPE(UN)
       /SAVE = FIXPRED(lunchpred)
       /TEST = "FR Lunch Between-School Effect" frlunch 1 SMfrlunch30 1.
CORRELATIONS /VARIABLES = math lunchpred.

 * STATA Model 3: Adding Fixed Effect of School Proportion Free/Reduced Lunch
xtmixed math c.frlunch c. SMfrlunch30, || schoolID: , ///
      variance ml covariance(un) residuals(independent),
      estat ic, n(94),
      predict lunchpred,            // save fixed-effect predicted outcomes
      estimates store FixFRLunch,   // save LL for LRT
      lincom 1*frlunch + 1*SMfrlunch30 // FR lunch between-school effect
corr math lunchpred                 // calculate total R2
```

STATA output:

```
                                                  Wald chi2(2)      =    926.41
Log likelihood = -54482.416                       Prob > chi2       =    0.0000

------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     frlunch |  -9.172883   .3344153   -27.43   0.000    -9.828325   -8.517441
 SMfrlunch30 |  -16.85017   2.000813    -8.42   0.000    -20.77169   -12.92865
       _cons |   50.60542   .4341687   116.56   0.000     49.75447    51.45638
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   13.48454   2.542895      9.317898    19.51437 → int var down by 49.85%
-----------------------------+------------------------------------------------
               var(Residual) |   239.3978   2.971595      233.6439    245.2935 → res var up by 0.03%
------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =   354.12 Prob >= chibar2 = 0.0000
.         estat ic, n(94),
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |     94           .   -54482.42      5     108974.8    108987.5
-----------------------------------------------------------------------------
               Note:  N=94 used in calculating BIC

.         predict lunchpred,             // save fixed-effect predicted outcomes
```

```
(option xb assumed)
.         estimates store FixFRLunch,                             // save LL for LRT

.         lincom 1*c.frlunch + 1*c.SMfrlunch30    // FR lunch between-school effect
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -26.02305   1.972668   -13.19   0.000    -29.88941    -22.1567
------------------------------------------------------------------------------
. corr math lunchpred                                           // calculate total R2
(obs=13082)
             |     math lunchp~d
-------------+------------------
        math |   1.0000
   lunchpred |   0.4038   1.0000
```

R = .4038, so total R^2 ~ .163

Total reduction from **both** lunch effects:
Intercept variance → 69.99% (of 15%)
Residual variance → 5.44% (of 85%)

What does the effect of school proportion free/reduced lunch represent in this model?
This is the contextual effect for FRlunch: holding child lunch status constant, for every 10% more children in your school who get free/reduced lunch, school mean math is lower by 1.69 points. Before controlling for individual lunch status, the reduction is 2.60 points per 10% (between-school effect, given in separate estimate).

What does the effect of student free/reduced lunch NOW represent in this model?
This is the pure within-school effect: holding school lunch status constant, children who receive free/reduced lunch score 9.17 points lower than children who don't.

Model 4: Adding a Random Effect of Student Free/Reduced Lunch (over Schools)

$$\text{Level 1: } \text{Math}_{ij} = \beta_{0j} + \beta_{1j}\left(\text{FRlunch}_{ij}\right) + e_{ij}$$
$$\text{Level 2: } \quad \text{Intercept: } \beta_{0j} = \gamma_{00} + \gamma_{01}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + U_{0j}$$
$$\text{Free/Reduced Lunch: } \beta_{1j} = \gamma_{10} + U_{1j}$$

```
TITLE1 "SAS Model 4: Adding Random Effect of Student Free/Reduced Lunch";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS schoolID studentID;
      MODEL math = frlunch SMfrlunch30 / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT frlunch / G TYPE=UN SUBJECT=schoolID; RUN;
```

```
TITLE "SPSS Model 4: Adding Random Effect of Student Free/Reduced Lunch".
MIXED math BY schoolID studentID WITH frlunch SMfrlunch30
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G
      /FIXED  = frlunch SMfrlunch30
      /RANDOM = INTERCEPT frlunch | SUBJECT(schoolID) COVTYPE(UN).
```

```
 * STATA Model 4: Adding Random Effect of Student Free/Reduced Lunch
xtmixed math c.frlunch c. SMfrlunch30, || schoolID: frlunch,  ///
      variance ml covariance(un) residuals(independent),
      estat recovariance, level(schoolID),
      estat ic, n(94),
      estimates store RandFRLunch      // save LL for LRT
      lrtest RandFRLunch FixFRLunch    // LRT against fixed effect model
```

STATA output:

```
                                                Wald chi2(2)       =    400.83
Log likelihood = -54438.694                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
```

```
      frlunch |      -8.45   .5611734   -15.06   0.000     -9.54988   -7.350121
  SMfrlunch30 |   -17.0879   1.917936    -8.91   0.000    -20.84698   -13.32881
        _cons |   50.25931   .5145964    97.67   0.000     49.25072     51.2679
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Unstructured       |
               var(frlunch) |   12.68699   3.311004      7.607035    21.15934 random slope var for frlunch
                 var(_cons) |   19.93184   3.745681      13.79068    28.80772 random intercept var
         cov(frlunch,_cons) |  -11.89358   3.164502     -18.09589   -5.691274 int-lunch covariance
-----------------------------+------------------------------------------------
              var(Residual) |   236.8373   2.946808      231.1316     242.684
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(3) =   441.57   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
.         estat ic, n(94),
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |     94           .   -54438.69      7     108891.4    108909.2
-----------------------------------------------------------------------------
               Note:  N=94 used in calculating BIC
.         estimates store RandFRLunch       // save LL for LRT
.         lrtest RandFRLunch FixFRLunch     // LRT against fixed effect model
Likelihood-ratio test                                 LR chi2(2)  =     87.45
(Assumption: FixFRLunch nested in RandFRLunch)        Prob > chi2 =    0.0000
Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter
space.  If this is not true, then the reported test is conservative.
```

Is model 4 better than model 3? *Yes*
−2ΔLL(2) = 87, p < .0001

So what does this mean about the effect of student free/reduced lunch?
The difference in math between kids who get free/reduced lunch and kids who don't varies significantly over schools.

95% random effects CI for the random FRlunch slope: → *-8.45 ± 1.96*SQRT(12.69) = −15.43 to −1.47*
On average, the gap related to lunch status is 8.45 points, but across 95% of the schools, that gap is predicted to be anywhere from 1.47 to 15.43 points.

Model 5: Adding a Cross-Level Interaction of Student by School Free/Reduced Lunch

$$\text{Level 1: } \text{Math}_{ij} = \beta_{0j} + \beta_{1j}\left(\text{FRlunch}_{ij}\right) + e_{ij}$$
$$\text{Level 2: } \quad \text{Intercept: } \beta_{0j} = \gamma_{00} + \gamma_{01}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + U_{0j}$$
$$\text{Free/Reduced Lunch: } \beta_{1j} = \gamma_{10} + \gamma_{11}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + U_{1j}$$

```
TITLE1 "SAS Model 5: Adding Cross-Level Interaction of Student by School Free/Reduced Lunch";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS schoolID studentID;
      MODEL math = frlunch SMfrlunch30 frlunch*SMfrlunch30 / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT frlunch / TYPE=UN SUBJECT=schoolID; RUN;

TITLE "SPSS Model 5: Adding Cross-Level Interaction of Student by School Free/Reduced Lunch".
MIXED math BY schoolID studentID WITH frlunch SMfrlunch30
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV
      /FIXED  = frlunch SMfrlunch30 frlunch*SMfrlunch30
      /RANDOM = INTERCEPT frlunch | SUBJECT(schoolID) COVTYPE(UN).

* STATA Model 5: Adding Cross-Level Interaction of Student by School Free/Reduced Lunch
xtmixed math c.frlunch c.smfrlunch30 c.frlunch#c.smfrlunch30, ///
      || schoolID: frlunch, variance ml covariance(un) residuals(independent),
      estat ic, n(94)
```

STATA output:

```
                                                Wald chi2(3)       =    413.76
Log likelihood = -54437.502                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------------------
                   math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
------------------------+-----------------------------------------------------------------
                frlunch |  -8.688252   .5673922   -15.31   0.000     -9.80032   -7.576183
           SMfrlunch30 |  -19.45931   2.473474    -7.87   0.000    -24.30723   -14.61139
c.frlunch#c.SMfrlunch30 |   4.140733   2.633721     1.57   0.116    -1.021265    9.302731
                  _cons |   50.22283   .5140769    97.70   0.000     49.21526     51.2304
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Unstructured       |
               var(frlunch) |   11.79733   3.165294      6.972708    19.96026 → slope var down by 7.01%
                 var(_cons) |   19.82708   3.701312      13.75171    28.58648 → int var down by 0.53%
         cov(frlunch,_cons) |  -11.34396   3.087016      -17.3944   -5.293523
-----------------------------+------------------------------------------------
              var(Residual) |   236.8234   2.946467      231.1183    242.6694 → res var down by .01%
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(3) =   442.87   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
.         estat ic, n(94)
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |     94           .    -54437.5      8       108891    108911.4
-----------------------------------------------------------------------------
```

What does the effect of student free/reduced lunch NOW represent in this model?
This is the difference between kids who get free/reduced lunch and those who don't in schools where 30% of the kids get free/reduced lunch: those kids who get free/reduced lunch are lower by 8.69.

What does the effect of school proportion free/reduced lunch NOW represent in this model?
This is the contextual (incremental between-school) effect for a kid who does not receive free/reduced lunch: for those kids, for every 10% more kids in their school that receive free/reduced lunch, their school mean math is lower by 1.94.

What does the cross-level interaction of student by school free/reduced lunch represent?
The effect of being a kid who receives free/reduced lunch is reduced nonsignificantly by 0.4 for every 10% more children in their school who get free/reduced lunch. But this effect is currently smushed—it assumes without testing that school FRlunch moderates the within-school and between-school effects of FRlunch to the same extent.

Model 6: Adding a Level-2 Interaction of Quadratic School Free/Reduced Lunch

$$\text{Level 1: } \text{Math}_{ij} = \beta_{0j} + \beta_{1j}\left(\text{FRlunch}_{ij}\right) + e_{ij}$$

$$\text{Level 2: } \quad \text{Intercept: } \beta_{0j} = \gamma_{00} + \gamma_{01}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + \gamma_{02}\left(\overline{\text{SchoolFRLunch}}_j - .30\right)^2 + U_{0j}$$

$$\text{Free/Reduced Lunch: } \beta_{1j} = \gamma_{10} + \gamma_{11}\left(\overline{\text{SchoolFRLunch}}_j - .30\right) + U_{1j}$$

```
TITLE1 "SAS Model 6: Adding Level-2 Interaction of Quadratic School Free/Reduced Lunch";
PROC MIXED DATA=work.grade10 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS schoolID studentID;
       MODEL math = frlunch SMfrlunch30 frlunch*SMfrlunch30 SMfrlunch30*SMfrlunch30
                    / SOLUTION DDFM=Satterthwaite OUTPM=work.TotalSave;
       RANDOM INTERCEPT frlunch / TYPE=UN SUBJECT=schoolID;
ESTIMATE "FR Lunch Between-School Main Effect" frlunch 1 SMfrlunch30 1;
ESTIMATE "FR Lunch Between-School Interaction" frlunch*SMfrlunch30 1 SMfrlunch30*SMfrlunch30 1;
RUN; PROC CORR NOSIMPLE DATA=work.TotalSave; VAR math pred; RUN;
```

```
TITLE "SPSS Model 6: Adding Level-2 Interaction of Quadratic School Free/Reduced Lunch".
MIXED math BY schoolID studentID WITH frlunch SMfrlunch30
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV
      /FIXED  = frlunch SMfrlunch30 frlunch*SMfrlunch30 SMfrlunch30*SMfrlunch30
      /RANDOM = INTERCEPT frlunch | SUBJECT(schoolID) COVTYPE(UN)
       /SAVE = FIXPRED(totalpred)
 /TEST = "FR Lunch Between-School Main Effect" frlunch 1 SMfrlunch30 1
 /TEST = "FR Lunch Between-School Interaction" frlunch*SMfrlunch30 1 SMfrlunch30*SMfrlunch30 1.
CORRELATIONS /VARIABLES = math totalpred.

* STATA Model 6: Adding Level-2 Interaction of Quadratic School Free/Reduced Lunch
xtmixed math c.frlunch c.SMfrlunch30 c.frlunch#c.SMfrlunch30 c.SMfrlunch30#c.SMfrlunch30, ///
      || schoolID: frlunch, variance ml covariance(un) residuals(independent),
      estat ic, n(94),
      predict totalpred,                                // save fixed-effect predicted outcomes
lincom 1*c.frlunch + 1*c.SMfrlunch30                    // FR lunch between-school main effect
lincom 1*c.frlunch#c.SMfrlunch30 + 1*c.SMfrlunch30#c.SMfrlunch30   // FR lunch BS interaction
margins, at(c.frlunch=(0 1) c.SMfrlunch30=(-.2 0 .2 .4)) vsquish     // create predicted values
marginsplot, noci name(predicted_lunch, replace) xdimension(frlunch)  // plot predicted, no CI
corr math totalpred                                                  // calculate total R2
```

STATA output:

```
                                                Wald chi2(4)      =    418.05
Log likelihood = -54436.242                     Prob > chi2       =    0.0000
--------------------------------------------------------------------------------------------
                        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------------------+---------------------------------------------------------------
                     frlunch |  -8.835737   .5769075   -15.32   0.000    -9.966455   -7.705019
                 SMfrlunch30 |  -17.98486   2.595472    -6.93   0.000    -23.07189   -12.89783
       c.frlunch#c.SMfrlunch30 |   5.428146   2.764887     1.96   0.050     .0090667    10.84723
c.SMfrlunch30#c.SMfrlunch30 |   -14.2013   8.815645    -1.61   0.107    -31.47965    3.077044
                       _cons |   50.85941   .6398308    79.49   0.000     49.60537    52.11346
--------------------------------------------------------------------------------------------

-----------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+-----------------------------------------------
schoolID: Unstructured       |
                var(frlunch) |   11.81828   3.178501      6.976308    20.02088 → slope var up by 0.18%
                  var(_cons) |   18.95016   3.572456      13.09621     27.4208 → int var down by 4.42%
           cov(frlunch,_cons) |  -10.92287   3.032665     -16.86678   -4.978956
-----------------------------+-----------------------------------------------
               var(Residual) |   236.8186   2.946416      231.1136    242.6645 → res var same
-----------------------------------------------------------------------------
LR test vs. linear regression:       chi2(3) =    426.87    Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
.         estat ic, n(94),
----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+--------------------------------------------------------------
           . |     94           .   -54436.24      9     108890.5    108913.4
----------------------------------------------------------------------------
               Note:  N=94 used in calculating BIC
```

What does the cross-level interaction of student by school free/reduced lunch NOW represent?
The effect of being a kid who receives free/reduced lunch (now after allowing for differential moderation across levels of the effects of free/reduced lunch at both levels by school mean free/reduced lunch) is reduced significantly by 0.54 for every 10% more children in their school who get free/reduced lunch.

What does the level-2 interaction of quadratic school free/reduced lunch represent?
*After controlling for kid free/reduced lunch status, the contextual (incremental between-school) effect of school mean free/reduced lunch as evaluated at 30% FRlunch becomes nonsignificantly more negative by 2*1.13 for every 10% more kids in their school with free/reduced lunch.*

```
. lincom 1*c.frlunch + 1*c.SMfrlunch30             // FR lunch between-school main effect
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   -26.8206   2.603258   -10.30   0.000    -31.92289   -21.71831
------------------------------------------------------------------------------
. lincom 1*c.frlunch#c.SMfrlunch30 + 1*c.SMfrlunch30#c.SMfrlunch30  // FR lunch between-school interaction
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -8.773157    8.41717    -1.04   0.297    -25.27051    7.724192
------------------------------------------------------------------------------
```

*If we don't control for kid free/reduced lunch, the between-school effect of −2.68 per 10% of school mean free/reduced lunch as evaluated at 30% FRlunch becomes nonsignificantly more negative by 2*0.88 for every 10% more kids in their school with free/reduced lunch.*

So school mean free/reduced lunch moderates the within-school FRlunch effect, but not the contextual (incremental between-school) or between-school effects.

```
. margins, at(c.frlunch=(0 1) c.SMfrlunch30=(-.2 0 .2 .4)) vsquish        // create predicted values
Adjusted predictions                              Number of obs   =      13082

Expression   : Linear prediction, fixed portion, predict()
1._at   : frlunch         =   0
          SMfrlunch30     = -.2
2._at   : frlunch         =   0
          SMfrlunch30     =   0
3._at   : frlunch         =   0
          SMfrlunch30     =  .2
4._at   : frlunch         =   0
          SMfrlunch30     =  .4
5._at   : frlunch         =   1
          SMfrlunch30     = -.2
6._at   : frlunch         =   1
          SMfrlunch30     =   0
7._at   : frlunch         =   1
          SMfrlunch30     =  .2
8._at   : frlunch         =   1
          SMfrlunch30     =  .4
```

```
------------------------------------------------------------------------------
     |            Delta-method
     |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+------------------------------------------------------------------------
 _at |
  1  |   53.88833   .6949427    77.54   0.000     52.52627     55.2504
  2  |   50.85941   .6398308    79.49   0.000     49.60537    52.11346
  3  |   46.69439   .7542279    61.91   0.000     45.21613    48.17265
  4  |   41.39326   1.296684    31.92   0.000      38.8518    43.93471
  5  |   43.96697    .884572    49.70   0.000     42.23324     45.7007
  6  |   42.02368   .5824752    72.15   0.000     40.88204    43.16531
  7  |   38.94428   .6012108    64.78   0.000     37.76593    40.12263
  8  |   34.72878   .9340579    37.18   0.000     32.89806     36.5595
------------------------------------------------------------------------------
```

```
. marginsplot, noci name(predicted_lunch, replace) xdimension(frlunch)  // plot predicted, no CI
  Variables that uniquely identify margins: frlunch SMfrlunch30

. corr math totalpred                                            // calculate total R2
(obs=13082)
             |     math totalp~d
-------------+------------------
        math |   1.0000
   totalpred |   0.4051   1.0000
```

R = .4051, so total R^2 = .164

Additional reduction from **both** interactions:
- Intercept variance → 4.93%
- Lunch slope variance → 6.85%
- Residual variance → 0.01%

Sample Results Section (note that "smushed" models are not reported)…

The extent to which student free/reduced lunch status could predict student math outcomes was examined in a series of multilevel models in which the 13,802 students were modeled as nested within their 94 schools. Maximum likelihood (ML) was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with individual Wald tests (i.e., of estimate / SE), whereas random effects were evaluated via likelihood ratio tests (i.e., −2ΔLL with degrees of freedom equal to the number of new random effects variances and covariances). Effect size was evaluated via pseduo-R^2 values for the proportion reduction in each variance component, as well as with total R^2, the squared correlation between the actual math outcomes and the math outcomes predicted by the fixed effects.

As derived from an empty means, random intercept model, student math scores had an intraclass correlation of .15, indicating that 15% of the variance in math scores was between schools. A 95% random effects confidence interval, calculated as fixed intercept ± 1.96*SQRT(random intercept variance), revealed that 95% of the sample schools were predicted to have intercepts for school mean math scores between 35 to 61. Children who did not receive free/reduced lunch were treated as the reference group. Given the large variability across schools in the proportion of students who received free/reduced lunch (0–80% of students), a contextual effect at level 2 was represented by the school proportion of students who receive free/reduced lunch centered near the sample mean of 30%.

The effects of free/reduced lunch status at each level were then added to the model. The within-school effect was significant and accounted for 5.44% of the residual variance, and indicated that students who receive free/reduced lunch are expected to have lower math scores than other students in their school by 9.18. The between-school effect was also significant and accounted for 70% of the remaining random intercept variance, and indicated that for every additional 10% of students who receive free/reduced lunch, that school's mean math score is expected to be lower by 2.60. After controlling for student free/reduced lunch, the contextual free/reduced lunch effect of −1.69 per additional 10% of students was still significant. A random slope for the effect of free/reduced lunch also resulted in a significant improvement in model fit, $-2\Delta LL(2) = 88.2$, $p < .001$, indicating that the size of the disadvantage related to free/reduced lunch differed significantly across schools. A 95% random effects confidence interval for the student free/reduced lunch effect, calculated as fixed slope ± 1.96*SQRT(random slope variance), revealed that 95% of the schools were predicted to have lunch-related gaps between students ranging from −15.45 to −1.46.

The extent to which school differences in the lunch-related disadvantage in math could be predicted from school lunch composition was then examined by adding a cross-level intra-variable interaction between the student and school lunch predictors, as well as the quadratic effect of school lunch composition to control for a contextual interaction effect. The within-school lunch effect was significantly moderated by school lunch composition (which reduced its random slope variance by 6.85%), although the moderation of the between-school and contextual effects was not significant, reducing the random intercept variance by another 4.93%, for a total $R^2 = .164$.

The significant intra-variable cross-level interaction, as shown by the nonparallel slopes of the lines in Figure 1, indicated that the lunch-related disadvantage in math scores of 8.84, as found for students receiving free/reduced lunch in schools in which 30% of students received free/reduced lunch, became significantly less negative by 0.54 for every additional 10% of students who received free/reduced lunch. Alternatively, the contextual school effect of −1.80 per 10% free/reduced lunch students (in baseline students in schools with 30% free/reduced lunch students) was reduced by 0.54 in free/reduced lunch students. The level-2 quadratic effect, seen by the widening distance between the lines in Figure 1, indicated that the same contextual school effect became nonsignificantly more negative by 1.42 for every additional 10% free/reduced lunch students (i.e., controlling for student lunch status), or that the between-school effect of −2.68 per 10% students became nonsignificantly more negative by 0.88 per 10% students (i.e., not controlling for student lunch status).

Figure 1: Plot of model-predicted math by free/reduced lunch status